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Generalized Scalar Duffin-Kemmer-Petiau Electrodynamics (GSDKP)

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Abstract. The main goal of this work is to investigate the quantum interaction between scalar field and gauge field in the context of Generalized Scalar Duffin-Kemmer-Petiau Electrodynamics (GSDKP) by a quantum theory in the functional approach. The Hamiltonian structure is obtained with the Dirac method and the Faddeev-Senjanovic procedure is established in order to write the transition amplitude in an alternative gauge fixing, known as the non-mixing gauge. As a consequence, the Schwinger-Dyson-Fradkin equations and the Ward-Takahashi-Fradkin identities are obtained.

1. Introduction

The purpose of this work is to study the covariant dynamics of systems with constraints and the quantization procedure in a particular case (GSDKP), so let us make a brief introduction to the subject.

The quantization problem of a classical dynamics was first put in a formal language by Dirac [1] because he observed that the classical dynamics described in the phase space by the observable and Poisson brackets was associated to a quantum dynamics described in the Hilbert space by the operators and commutators\anti-commutators via correspondence principle

$$\{,\}_P \to -i[,]_\pm. \tag{1}$$

Later the existence of constraints in the hamiltonian dynamics led Dirac to extend its mechanical analysis of the phase space by the parenthesis of Dirac and the classification (first class\second class) always being able to make connection to quantum dynamics with the correspondence principle [2],

$$\{,\}_D \to -i[,]_\pm. \tag{2}$$

This is the first look at the connection between classical\quantum dynamics.

The second look begins in a Dirac study [3] about the connection between a dynamics described in the configuration space and its resulting quantum dynamics. In this study we see the emergence of an object very important called transition amplitude

$$Z = \int D\mu \exp[iS], \tag{3}$$

where $D\mu$ is the integration measure and S is the action.

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Feynman uses the idea of Dirac to formulate a way to describe the quantum mechanics via path integral [4]. However elegantly Schwinger infers that because the quantum equations in the Heisenberg description preserve its classic form it should be a principle of quantum action [5]. We have now that the amplitude transition, or functional generator is a solution of the functional equation arising from a dynamic

$$[\dot{\varphi} - \{\varphi, H_0 - \varphi J\}_D]|_{\varphi=\frac{\delta}{i\delta J}} Z[J] = 0 \tag{4}$$

where H_0 is the initial hamiltonian.

Continuing, Faddeev explores the integration measure properties $D\mu$ to study connections between classical\quantum dynamics of the physical systems with first-class constraints [6]. Later Senjanovic extends the ideas of Faddeev [7] for second-class constraints. Finally Fradkin and Vilkovisky make an overview of the problem in view of covariant formalisms and its connection with BRST symmetry [8]. See, for more details on quantization of systems with constraints [9].

2. Constraint analysis and transition amplitude

To construct the transition amplitude we will use the Faddeev-Senjanovic [6, 7] analysis where we must first do a short study of constraint.

The Lagrangian density describing the GSDKP is defined by

$$\mathscr{L} = \frac{i}{2}\bar{\psi}\beta^{\mu}\left(\partial_{\mu}\psi\right) - \frac{i}{2}\left(\partial_{\mu}\bar{\psi}\beta^{\mu}\right)\psi - m\bar{\psi}\psi + eA_{\mu}\bar{\psi}\beta^{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}\partial^{\mu}F_{\mu\beta}\partial_{\alpha}F^{\alpha\beta} \tag{5}$$

where $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the usual electromagnetic field-strength tensor and β^{μ} are the DKP matrices that obey the algebra [10]

$$\beta^{\mu}\beta^{\nu}\beta^{\theta} + \beta^{\theta}\beta^{\nu}\beta^{\mu} = \beta^{\mu}\eta^{\nu\theta} + \beta^{\theta}\eta^{\nu\mu}\ . \tag{6}$$

This theory is invariant under local gauge transformations

$$\psi \to e^{i\alpha(x)}\psi, \quad A_{\mu} \to A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha(x)\,. \tag{7}$$

The matter (scalar field, mesons) are described by the Duffin-Kemmer-Petiau (DKP) theory [11], a first-order relativistic theory for the description of spin 0 and spin 1 bosons with a similar form as the Dirac equation, and the radiation (vector field, photons) are described by the Podolsky theory [12]. We can see this gauge theory with U(1) symmetry as an extension of the work on DKP Quantum Electrodynamics [13].

The translational space-time invariance of the Lagrangian density leads us to write the canonical Hamiltonian

$$\begin{aligned} H_c = \int d^3x \Big[&(\partial_0\bar{\psi})\frac{\partial\mathscr{L}}{\partial(\partial_0\bar{\psi})} + \frac{\partial\mathscr{L}}{\partial(\partial_0\psi)}(\partial_0\psi) + \frac{\partial\mathscr{L}}{\partial(\partial_0 A_{\nu})}(\partial_0 A_{\nu}) \\ &- \partial_{\theta}\left(\frac{\partial\mathscr{L}}{\partial(\partial_0\partial_{\theta}A_{\nu})}\right)(\partial_0 A_{\nu}) + \frac{\partial\mathscr{L}}{\partial(\partial_0\partial_{\theta}A_{\nu})}(\partial_{\theta}\partial_0 A_{\nu}) - \mathscr{L}\Big], \end{aligned} \tag{8}$$

where we define the momenta expressions

$$p = \frac{\partial\mathscr{L}}{\partial(\partial_0\bar{\psi})} = -\frac{i}{2}\beta^0\psi, \tag{9}$$

$$\bar{p} = \frac{\partial\mathscr{L}}{\partial(\partial_0\psi)} = \frac{i}{2}\bar{\psi}\beta^0, \tag{10}$$

$$\Pi^{\nu} = F^{\nu 0} + a^2[\eta^{i\nu}\partial_i\partial_{\alpha}F^{\alpha 0} - \partial_0\partial_{\alpha}F^{\alpha\nu}], \tag{11}$$

$$\Phi^{\nu} = a^2[\partial_{\alpha}F^{\alpha\nu} - \eta^{\nu 0}\partial_{\alpha}F^{\alpha 0}]. \tag{12}$$

From the above momentum expressions, we shall study the constraint structure of the theory following Dirac's approach to singular systems [2]. In this approach it is possible to obtain the set of first-class constraints

$$\varphi_1 = \Phi_0 \approx 0, \tag{13}$$
$$\varphi_2 = \Pi_0 - \partial_k \Phi^k \approx 0, \tag{14}$$
$$\varphi_3 = e\bar{\psi}\beta^0\psi - \partial^k \Pi_k \approx 0 \tag{15}$$

and the set of second-class constraints

$$\chi^{(1)} = p + \frac{i}{2}\beta^0\psi \approx 0, \tag{16}$$
$$\bar{\chi}^{(1)} = \bar{p} - \frac{i}{2}\bar{\psi}\beta^0 \approx 0, \tag{17}$$
$$\chi^{(2)} = [1-(\beta^0)^2][i\beta^i\partial_i\psi(x) - m\psi(x) + e\beta^i A_i(x)\psi(x)] \approx 0, \tag{18}$$
$$\bar{\chi}^{(2)} = [-i\partial_i\bar{\psi}(x)\beta^i + m\bar{\psi}(x) - e\bar{\psi}(x)\beta^i A_i(x)][1-(\beta^0)^2] \approx 0. \tag{19}$$

The weak equality $\approx$ is understood in according to Dirac's sense.

With the full set of first-class and second-class constraints determined, we are now in a position to obtain the functional generator. The transition amplitude in the Hamiltonian form is written in the following way:

$$Z = N\int D\mu \exp\left\{ i\int d^4x [(\partial_0\bar{\psi})\, p + \bar{p}\,(\partial_0\psi) + \Pi^\nu\,(\partial_0 A_\nu) + \Phi^\nu\,(\partial_0\Gamma_\nu) - \mathscr{H}_c]\right\} \tag{20}$$

where the canonical Hamiltonian is given by

$$\begin{aligned}\mathscr{H}_c = {} & \Pi_0\Gamma^0 + \Pi_k\Gamma^k + \Phi_k(\partial^k\Gamma_0 - \partial_l F^{lk} + \frac{\Phi^k}{2a^2}) - \frac{i}{2}\bar{\psi}\beta^i \overleftrightarrow{\partial}_i\psi + m\bar{\psi}\psi \\ & - e\bar{\psi}\hat{A}\psi + \frac{1}{4}F_{kj}F^{kj} + \frac{1}{4}(\Gamma_j - \partial_j A_0)^2 - \frac{a^2}{2}(\partial^j\Gamma_j - \partial^j\partial_j A_0)^2,\end{aligned} \tag{21}$$

and the integration measure

$$D\mu = D\Phi^\nu D\Gamma_\nu D\Pi^\mu DA_\mu D\bar{\psi}D\psi D\bar{p}Dp\,\delta(\Theta_l)\det\|\{\Theta_l,\Theta_m\}\|^{\frac{1}{2}}. \tag{22}$$

We have that the complete set of constraints for the GSDKP is

$$\Theta_l = \left\{\chi^{(1)}, \bar{\chi}^{(1)}, \chi^{(2)}, \bar{\chi}^{(2)}, \varphi_1, \varphi_2, \varphi_3, \Sigma_1, \Sigma_2, \Sigma_3\right\}, \tag{23}$$

in which suitable gauge conditions for the first-class constraints are chosen as the generalized radiation conditions [14]

$$\Sigma_1 = \Gamma_0(x) \approx 0, \quad \Sigma_2 = A_0 \approx 0, \quad \Sigma_3 = (1 + a^2\Box)(\vec{\nabla}.\vec{A}) \approx 0. \tag{24}$$

After integrating over the gauge and fermionic momenta, the transition amplitude Z is explicitly written

$$\begin{aligned} Z = N\int & DA_\mu D\bar{\psi}D\psi \det\left\|(1+a^2\vec{\nabla}^2)\vec{\nabla}^2\right\|\delta((1+a^2\Box)(\vec{\nabla}.\vec{A})) \\ & \exp[i\int d^4x\{\bar{\psi}(i\beta^\mu\nabla_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}\partial^\mu F_{\mu\beta}\partial_\alpha F^{\alpha\beta}\}].\end{aligned} \tag{25}$$

Hence, with the Faddeev-Popov-DeWitt ansatz in the non-mixing gauge condition [15]

$$\Omega(A) = \left(1 + a^2\Box\right)^{\frac{1}{2}} \partial^\mu A_\mu \tag{26}$$

the transition amplitude can be written in a covariant form

$$\begin{aligned} Z = N \int DA_\mu D\bar{\psi} D\psi \exp\{i \int d^4x [\bar{\psi}\left(i\beta^\mu \nabla_\mu - m\right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ + \frac{a^2}{2} \partial^\mu F_{\mu\beta} \partial_\alpha F^{\alpha\beta} - \frac{1}{2\xi}\left(\partial^\mu A_\mu\right)\left(1 + a^2\Box\right)\left(\partial^\mu A_\mu\right)]\}. \end{aligned} \tag{27}$$

Although initially Podolsky used the Lorenz gauge fixing condition $\Omega(A) = (\partial_\mu A^\mu)$ to fix the physical degrees of freedom in generalized electrodynamics, lately with a rigorous study of constraint analysis we see that this is not really true [14]. As a consequence, the natural way to fix the physical degrees of freedom was the generalized Lorenz gauge fixing condition $\Omega(A) = (1 + a^2\Box)(\partial_\mu A^\mu)$, however it complicates the theory's quantization once it increases the order of the field equation. Today we think that the non-mixing gauge fixing condition [15] $\Omega(A) = \left(1 + a^2\Box\right)^{\frac{1}{2}} \partial^\mu A_\mu$ combines with the theory because it preserves the order of the field equation, but it has a pseudo-differential structure.

3. The Schwinger-Dyson-Fradkin equations

The most elegant way of studying the set of field equations in the Heisenberg description is in a functional formulation, consisting in an infinite chain of differential equations that relate different Green's function in a exact manner. This infinite tower of equations is referred to as the Schwinger-Dyson-Fradkin (SDF) equations [17].

3.1. The Schwinger-Dyson-Fradkin equations for the gauge-field propagator

The complete expression of the gauge-field propagator can be determined by means of the functional generator leading to the Schwinger variational equation for the gauge field,

$$\left[\left. \frac{\delta S}{\delta A_\gamma(x)} \right|_{\frac{\delta}{\delta i\eta}, \frac{\delta}{\delta i\bar{\eta}}, \frac{\delta}{\delta i J_\mu}} + J^\gamma(x) \right] \mathscr{Z}\left[\eta, \bar{\eta}, J_\mu\right] = 0. \tag{28}$$

In terms of the functional $\mathscr{Z}[J] = \exp\{iW[J]\}$ for the connected Green's functions we obtain the equation

$$\begin{aligned} -J^\gamma(x) = &- ie \frac{\delta}{\delta\eta(x)} \beta^\gamma \left(\frac{\delta W}{\delta\bar{\eta}(x)}\right) + e \frac{\delta W}{\delta\eta(x)} \beta^\gamma \frac{\delta W}{\delta\bar{\eta}(x)} \\ &+ \left[T^{\gamma\mu} + \frac{1}{\xi} L^{\gamma\mu}\right]\left(1 + a^2\Box\right)\Box \frac{\delta W}{\delta J^\mu(x)}. \end{aligned} \tag{29}$$

The last equation can be interpreted as the complete Podolsky field equation subjected to an external source J^γ. On this equation $T^{\gamma\mu}$ and $L^{\gamma\mu}$ are differential projectors

$$T^{\gamma\mu} + L^{\gamma\mu} = g^{\gamma\mu}, \quad L^{\gamma\mu} = \frac{\partial^\gamma \partial^\mu}{\Box}. \tag{30}$$

In order to obtain the complete gauge-field propagator it proves convenient to introduce also the generating functional for the one particle irreducible (1PI) Green's functions, which is related to W by a functional Legendre transformation

$$\Gamma\left[\psi, \bar{\psi}, A_\mu\right] = W\left[\eta, \bar{\eta}, J_\mu\right] - \int d^4x \left(\bar{\psi}\eta + \bar{\eta}\psi + A^\mu J_\mu\right). \tag{31}$$

Hence, rewriting (29) in terms of the 1PI functional

$$\frac{\delta^2\Gamma}{\delta A_\nu(y)\,\delta A_\gamma(x)} = \left[T^{\gamma\nu} + \frac{1}{\xi}L^{\gamma\nu}\right]\left(1+a^2\Box\right)\Box\delta^{(4)}(x,y) + ie^2\int d^4u d^4w Tr\left[\mathscr{S}(x,u;A)\,\beta^\gamma\mathscr{S}(w,x;A)\,\Gamma^\nu(u,w;y)\right]. \tag{32}$$

The second term of (32) can be identified with the polarization operator, $\mathfrak{P}^{\gamma\nu}$,

$$\mathfrak{P}^{\gamma\nu}(x,y) = ie^2\int d^4u d^4w Tr\left[\mathscr{S}(x,u;A)\,\beta^\gamma\mathscr{S}(w,x;A)\,\Gamma^\nu(u,w;y)\right]. \tag{33}$$

The expression for the inverse of the gauge-field complete propagator in momentum representation is

$$\left(\mathfrak{D}^{\gamma\nu}\right)^{-1}(p) = \left(D^{\gamma\nu}\right)^{-1}(p) + \mathfrak{P}^{\gamma\nu}(p). \tag{34}$$

This equation can be viewed as

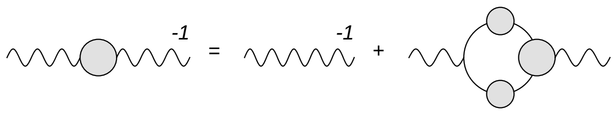

Figure 1. The SDF equation for the gauge-field propagator.

The expression (34) can be solved in order to find

$$i\mathfrak{D}^{\gamma\nu}(p) = -\frac{\left(\eta^{\gamma\nu} - \frac{p^\gamma p^\nu}{p^2}\right)}{\left[\mathfrak{P}(p) - (1-a^2p^2)\,p^2\right]} + \frac{\xi}{p^2(1-a^2p^2)}\frac{p^\gamma p^\nu}{p^2}, \tag{35}$$

in which the scalar polarization $\mathfrak{P}(p)$ it is related to the polarization tensor $\mathfrak{P}^{\gamma\nu}(p)$, through the structure

$$\mathfrak{P}^{\gamma\nu}(p) = \left(-p^2\eta^{\gamma\nu} + p^\gamma p^\nu\right)\mathfrak{P}(p). \tag{36}$$

For the free propagator, namely taking $\mathfrak{P}(p)=0$ on (35) and $a = m_p^{-1}$, one has the expression

$$iD^{\gamma\nu}(p) = \left[\eta^{\gamma\nu} - (1-\xi)\frac{p^\gamma p^\nu}{m_p^2}\right]\left[\frac{1}{p^2} - \frac{1}{p^2-m_p^2}\right] - (1-\xi)\frac{p^\gamma p^\nu}{(p^2)^2} \tag{37}$$

3.2. The Schwinger-Dyson-Fradkin equations for the DKP propagator

The expression to the full DKP propagator can be derived starting from Schwinger variational equation

$$\left[\left.\frac{\delta S}{\delta\bar{\psi}(x)}\right|_{\frac{\delta}{\delta i\eta},\frac{\delta}{\delta i\bar{\eta}},\frac{\delta}{\delta iJ_\mu}} + \eta(x)\right]\mathscr{Z}\left[\eta,\bar{\eta},J_\mu\right] = 0. \tag{38}$$

Now, writing the equation (38) in terms of the generating functional W and then differentiating the resulting expression with respect to the source $\eta(y)$, we obtain

$$i\delta^{(4)}(x-y) = -\left[i\beta^\mu\partial_\mu - m + e\beta^\mu\langle A^\mu\rangle - ie\beta^\mu\frac{\delta}{\delta J_\mu(x)}\right]\mathscr{S}(x,y;A). \tag{39}$$

By solving the derivative of the last term we can identify

$$\Sigma(x,z) = ie^2\beta^\mu \int d^4u d^4v \mathfrak{D}_\mu^\alpha(u,x)\mathscr{S}(x,v;A)\Gamma_\alpha(v,z;u), \tag{40}$$

as the DKP self-energy function Σ. Hence, by taking the limit of null sources, we find that

$$i\delta^{(4)}(x-y) = -\left[i\beta^\mu\partial_\mu - m\right]\mathscr{S}(x,y;A) + \int d^4z\Sigma(x,z)\mathscr{S}(z,y;A). \tag{41}$$

In momentum representation the above equation becomes $\mathscr{S}^{-1}(p) = S^{-1}(p) + \Sigma(p)$. The later equation can be viewed as

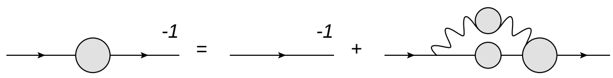

Figure 2. The SDF equation for the scalar propagator.

The equation (39) can formally be written as

$$\mathscr{S}(p) = \frac{i}{\beta^\mu p_\mu - \mathfrak{M}(p)} \tag{42}$$

where we defined the mass operator $\mathfrak{M}$,

$$\mathfrak{M}(p) = m + \Sigma(p). \tag{43}$$

Besides, the expression for the DKP free propagator can be obtained with help of the DKP algebra (6),

$$S(p) = i\frac{1}{m}\left[\frac{\hat{p}(\hat{p}+m)}{(p^2-m^2)} - 1\right], \quad \hat{p} = \beta^\mu p_\mu. \tag{44}$$

3.3. The Schwinger-Dyson-Fradkin equations for the vertex part

The starting point to get the vertex function is the equation (39). In a similar way, we take the derivative of the resulting expression with respect to the field $A_\sigma(z)$, and after some manipulations, we find the following expression for the vertex function

$$i\Gamma^\sigma(q,p;k) = -\beta^\sigma(2\pi)^4\delta(q-p-k) + i\Lambda^\sigma(k,p;q) \tag{45}$$

where we have introduced a new quantity, the vertex part

$$i\Lambda^\sigma(q,p;k) = ie^2\beta^\mu\frac{1}{(2\pi)^4}\int d^4t\mathfrak{D}_{\mu\rho}(t)\mathscr{S}(t+k)\Phi^{\sigma\rho}(t+k,p;q,t) +$$
$$+e^2\beta^\mu\frac{1}{(2\pi)^4}\int d^4p_1d^4p_2\mathfrak{D}_{\mu\rho}(p_1)\mathscr{S}(p_1+k)\Gamma^\sigma((p_1+k),p_2;q)\mathscr{S}(p_2)\Gamma^\rho(p_2,p;p_1), \tag{46}$$

in this expression we have defined the four-point vertex function

$$e^2\Phi^{\sigma\rho}(a,w;z,s) = \frac{\delta^4\Gamma}{\delta A_\rho(s)\delta A_\sigma(z)\delta\psi(w)\delta\bar{\psi}(a)}. \tag{47}$$

From Eq.(46) we clearly see that the three-point vertex function depends on the four-point one.

Diagrammatically, the irreducible vertex part can be visualized

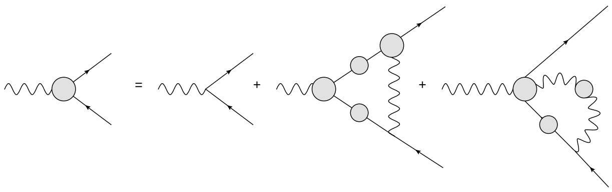

Figure 3. The SDF equation for the vertex function.

4. Ward-Takahashi-Fradkin identities

It is of our understanding that when we formulate the equations of electrodynamics in a covariant way we can describe them by a Lagrangian that has at its cornerstone a gauge symmetry. Classically the matter field (scalar) has a global U(1) gauge symmetry. When we impose a local gauge symmetry we need intermediaries fields of interaction (vector). Therefore when we write the dynamics of this electrodynamics (GSDKP) in a covariant way we have a local gauge symmetry U(1). Similarly we can describe the equations of quantum electrodynamics in a covariant way in the Heisenberg representation. On the other hand when we quantize the theory covariantly using the functional formalism we lost the gauge symmetry, to fix the physical degrees of freedom and define adequately the measure of integration $\int D\mu$. To maintain gauge symmetry we impose it in the functional generator and therefore this generates certain constraints on Green's functions, relationships between them known as Ward-Takahashi-Fradkin identities (WTF) [18]. The gauge symmetry is a consequence of a covariant quantum dynamics, the symmetry is implicit in the quantum equations of motion discussed previously.

The derivation of the WTF identities is formally given in terms of the following identity upon the functional generator

$$\left.\frac{\delta\mathscr{Z}\left[\eta,\bar{\eta},J_\mu\right]}{\delta\alpha(x)}\right|_{\alpha=0}=0. \tag{48}$$

This leads to the equation of motion satisfied by $\mathscr{Z}\left[\eta,\bar{\eta},J_\mu\right]$

$$\left[-i\frac{\Box}{e\xi}\left(1+a^2\Box\right)\partial^\mu\frac{\delta}{\delta J^\mu}-\frac{\delta}{\delta\eta}\eta+\bar{\eta}\frac{\delta}{\delta\bar{\eta}}-\frac{1}{e}\partial^\mu J_\mu\right]\mathscr{Z}=0. \tag{49}$$

Finally, one can get the desired quantum equation of motion for the theory by writing (49) first in terms of W, and then as an expression for the 1PI-generating functional $\Gamma\left[\psi,\bar{\psi},A_\mu\right]$ through the relation (31). Then

$$-i\frac{\Box}{e\xi}\left(1+a^2\Box\right)\partial_x^\mu A_\mu-\bar{\psi}\frac{\delta\Gamma}{\delta\bar{\psi}}+\frac{\delta\Gamma}{\delta\psi}\psi+\frac{1}{e}\partial_x^\mu\frac{\delta\Gamma}{\delta A^\mu}=0. \tag{50}$$

This is the equation that will supply all the WFT identities.

The first identity comes by applying the derivatives of (50) with respect to $\psi(y)$ and $\bar{\psi}(z)$, yielding

$$\partial^\mu\Gamma_\mu(z,y;x)=-\delta(x-z)\Gamma(x,y)+\Gamma(x,z)\delta(x-y). \tag{51}$$

In momentum representation

$$k_\mu\Gamma^\mu\left(p,p',k=p-p'\right)=\mathscr{S}^{-1}\left(p'\right)-\mathscr{S}^{-1}(p). \tag{52}$$

Furthermore, we may consider the limit of this equation as $k\to 0$, in which we find that the vertex part is related to the DKP self-energy function as

$$\Lambda^\mu(p,p,k=0)=-\frac{\partial}{\partial p_\mu}\Sigma^{-1}(p). \tag{53}$$

On the other hand, upon the differentiation of (50) with respect to $A_\nu(y)$, it follows the identity

$$\partial_\mu \Gamma^{\mu\nu}(x,y) = \frac{\Box}{\xi}\left(1+a^2\Box\right)\partial^\nu \delta^{(4)}(x-y) \tag{54}$$

which together with equation (33) implies that

$$k_\mu \mathfrak{P}^{\mu\nu}(k) = 0. \tag{55}$$

5. Conclusions

In this work we have analyzed the covariant dynamics of interaction between scalar particles and generalized photons. The first point to note is that we have implemented the non-mixing gauge in the constraint analysis. The second point to note is that we have used the DKP field to describe the scalar particles. The complete quantum structure of the scalar field, seen by means of SDF diagrams, is exactly as those from $GQED_4$. In this case the same diagram phenomenology for the electromagnetic interaction between scalar or fermionic fields happens. But the DKP fields are described by the DKP algebra (mesonic algebraic structure), while the fermionic field obeys a Clifford algebra (fermionic algebraic structure). This work complements previous studies on covariant quantum dynamics [19] and opens the door to a more complete approach, with the study of renormalization, first radiative corrections and quantum covariant dynamics in equilibrium (Matsubara-Fradkin formalism).

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