DESCRIPTION OF HEIGHT GROWTH OF HYBRID EUCALYPTUS CLONES IN SEMI-ARID REGION USING NON-LINEAR MODELS

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- ABSTRACT: Brazil stands out worldwide for planting homogeneous forests, mainly pine and eucalyptus. Forestry production is of great importance for the country's economy, being also a reference in sustainability, competitiveness and innovation. Of the 10 million hectares of planted trees, 76.3% is composed of the genus *Eucalyptus,* which makes Brazil one of the largest producers of this genus in the world. The analysis of the growth trajectory of trees of this genus can be a great ally in improving the management plans currently used. In this sense, the aim of this study was to compare the performance of the nonlinear models Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart, which were fit using the R software considering the first order autoregressive error structure (AR1), applied to data of average height, in meters, in relation to time, in months, totaling 15 observations obtained during six and a half years. Nonlinearity measures were used to check the adequacy of the linear approximations of models and as criteria for model selection the R^2, AIC_C and DPR, with the Schöngart (AR1) model being the one that best fit the data.

- KEY WORDS: Growth curve; autoregressive errors; regression; Schöngart.

1 Introduction

Brazil stands out worldwide for its extensive forest cover, sheltering the second largest forest area on the planet, which, according to the Brazilian Forest Service (BRASIL, 2020), is equivalent to more than half of its territory (55%). According to the last report presented by FAO (2020), most of this cover is composed of natural forests, but the country also stands out with the planting of homogeneous forests, mainly of the genera *Pinus* and *Eucalyptus*.

The Brazilian planted tree industry is recognized worldwide for its high forest production, which, according to the latest report by the Brazilian Tree Industry (IBÁ, 2020),

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occupied a prominent position among the top ten in the global ranking of forest productivity in 2019, which shows the great importance of this sector for the country's economy, being also a reference in sustainability, competitiveness and innovation. It is estimated that in 2019, the total number of jobs in the forest sector was around 3.8 million, generating income in all regions of the country and accounting for 1.2% of the National GDP.

Brazil has 10 million hectares of planted trees with about 76.3% of this area occupied by *Eucalyptus*, thus being one of the largest producers of this genus in the world (IBGE, 2020). According to Lima Filho *et al*. (2012), eucalyptus stands out for its ease of reproduction, rusticity, wood quality, high level of genetic improvement in productivity and for its high growth rate.

In Forestry Sciences, the use of models to understand the growth of trees in a stand is of paramount importance, being widely applied to native and commercially exploited species aiming at the sustainability of cutting cycles (SCHÖNGART, 2008; MIRANDA *et al.*, 2017; ANDRADE *et al.*, 2019; DAVID *et al.*, 2019). Likewise, the application of modeling can help the development of planted forests, as this makes it possible to choose appropriate silvicultural methods and sustainable forest management plans to be adopted, which can increase planting productivity (MACHADO *et al.*, 2012; SILVA *et al.*, 2017; McEWAN *et al.*, 2019).

Tree growth is usually best described by nonlinear models and this is due to the fact that their growth is generally faster in its initial phase, decreasing its speed over time and tending to stability in the adult phase, which can be well represented by sigmoidal growth curves (CORDEIRO *et al.*, 2009; PRADO *et al.*, 2013; MANGUEIRA *et al.*, 2016; MISCHAN and PINHO, 2014). Several studies have been carried out using nonlinear models to describe the growth of planted trees, such as the works by Silva *et al*. (2017) for data from eucalyptus clones, Coutinho *et al.* (2017) in relation to the diameter of *Cryptomeria japonica* and also for native species of tropical and subtropical forests and Cerrado throughout the Brazilian territory (FIGUEIREDO FILHO *et al.*, 2017; BARBOSA *et al.*, 2018; GRANATO-SOUZA *et al.*, 2019; DAVID *et al.*, 2019).

Nonlinear models have some advantages over linear models, such as parsimony and practical and direct interpretation of their parameters (SANTOS *et al.*, 2013; LIMA *et al.*, 2017; JANE *et al.*, 2019). However, as in the estimation of parameters by the least squares method, system of normal equations has no solution, iterative methods must be used. There are several iterative methods in the literature, in which the Gauss Newton method os the most used (ARCHONTOULIS; MIGUEZ, 2015; FERNANDES *et al.*, 2015; FURTADO *et al.*, 2019; SILVA *et al.*, 2019).

It is important in the adjustment of regression models that some assumptions are met as the independence of the residual vector, but when this assumption is not met, it is necessary to incorporate an autoregressive error structure to obtain more accurate estimates of the studied model parameters, as can be seen in Souza *et al*. (2014) and Prado *et al*. (2020), who used the structure of first-order autoregressive errors with satisfactory results to describe the germination of coffee seeds and the growth of the internal cavity of green dwarf coconut fruits, respectively.

Considering the importance of nonlinear models in forestry studies, this study aimed to describe the growth in height of *Eucalyptus* hybrid clones, planted at the Experimental Station of the Agronomic Institute of Pernambuco, using nonlinear regression models. The hypothesis that guides this study is that the growth in height of hybrid clones of the genus *Eucalyptus* is directly related to age and can be described by nonlinear models.

2 Material and methods

Data used were obtained from an experiment described by Lima Filho *et al.* (2012), who used 15 clones of the genus *Eucalyptus* in an area of 2.5 hectares, which was established in March 2002 at the Gesseiro do Araripe Pole at the Experimental Station of the Agronomic Institute of Pernambuco in Chapada do Araripe, state of Pernambuco, a region with predominant hot and dry climate from low latitudes, with summer rains. The average temperature is 24.6ºC, with a maximum of 33ºC and a minimum of 15.9ºC and total annual rainfall around 735 mm. The soil is red-yellow latosol.

The experiment was installed in a region at the geographic coordinates 07º29'00 S and 40º36'00 W and 816 m altitude, where 100 trees of one of the clones of the hybrid *Eucalyptus urophylla* X *Eucalyptus tereticornis* X *Eucalyptus pellita*, were planted with four repetitions of 25 trees in each. Seedlings of *Eucalyptus* clones came from Comercial Agrícola Paineiras Ltda., located in Urbano Santo, state of Maranhão.

For model adjustment, the mean height measured over time was used in the 83 surviving trees, during six and a half years. The first measurement was carried out at two months, the age at which the seedlings were planted in the field, from the second measurement onwards, an interval of six months was respected.

Parameter estimates were found by adjusting five nonlinear models for mean height (m) as a function of age (months): Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart (Table 1).

Table 1 - Nonlinear models used to describe the growth of the *Eucalyptus* hybrid clone

Model	Functional form	Reference
Gompertz	$y_i = \alpha e^{-e^{\kappa(\beta - t_i)}} + u_i$	FERNANDES *et al.* (2015)
von Bertalanffy	$y_i = \alpha \left(1 - e^{\frac{\kappa(\beta - t_i)}{3}}\right)^3 + u_i$	FERNANDES *et al.* (2020)
Brody	$y_i = \alpha \left[1 - \beta e^{(-\kappa . t_i)}\right] + u_i$	RIBEIRO *et al.* (2018a)
Chapman-Richards	$y_i = \alpha \left(1 - e^{-\kappa t_i}\right)^{\beta} + u_i$	LIMA FILHO *et al.* (2012)
Schöngart	$y_i = \dfrac{\alpha}{1 + (\beta / t_i)^{\kappa}} + u_i$	SCHÖNGART (2008)

For equations in Table 1, y_i represents the i-th observation of the average height, in meters, observed at the time of measurement t_i, in months, with *i* ranging from 1 to 15; α is the asymptotic value, that is, the expected value for the maximum height of the trees under study; β is the abscissa of the inflection point for the Gompertz and von Bertalanffy models, that is, growth is decelerated from it and for the Brody, Chapman-Richards and Schöngart models it has no practical interpretation due to the characteristics of its curve; κ is an index of maturity or precocity, it is associated with growth; u_i is the residual of the fit at the ith time.

Due to the residual dependence, it was necessary to consider the first-order autoregressive models, in which the u_i errors are expressed in $u_i = \phi_1 u_{i-1} + \varepsilon_i$, where ϕ_1 is the order 1 autoregressive parameter; u_{i-1} is the residual of the time adjustment immediately before the i-th measurement and ε_i is the so-called white noise, with normal distribution, zero mean and constant variance σ^2, that is, $\varepsilon_i \sim N(0, \sigma^2)$.

To check the assumptions of homoscedasticity, normality and independence of residuals, the Breusch-Pagan, Shapiro-Wilk and Durbin-Watson tests were used, respectively, at the 1% level of significance, and in addition, graphic analyses of the residuals were performed. To estimate the parameters of the models, the least squares method was applied and the Gauss-Newton convergence algorithm was used as an iterative method, using the "*Manipulate*" package as a tool to obtain the initial values.

The adequacy of linear approximations upon parameter estimation was observed through the nonlinearity measures and the selection of the model that best fit the data was made based on the coefficient of determination (R^2), corrected Akaike's information criterion (AIC_C) and residual standard deviation (RSD). The estimation of the model parameters, their graphs, analysis of residuals, confidence intervals and the verification of the goodness of fit for model selection were performed using the R statistical software (R CORE TEAM, 2020), free access, using the "*nlme*", "*car*", "*lmtest*" and "*qpcR*" packages.

3. RESULTS AND DISCUSSION

Table 2 lists the results obtained for the Breusch-Pagan, Durbin-Watson and Shapiro-Wilk tests used to check the assumptions of homoscedasticity, independence and normality of residuals, respectively. To check normality and constant variance, the tests were not significant (p-value > 0.01), so the residuals were normally distributed with constant variance. Regarding the Durbin-Watson test, it presented significant values (p-value < 0.01) for the residuals in all adjusted models, indicating residual autocorrelation, which was expected because data were obtained in the same tree over time, which corroborates Cassiano and Sáfadi (2015), who claim that observations ordered over time and taken from the same individual are generally autocorrelated.

Table 2 - P-value for the Breusch-Pagan, Shapiro-Wilk and Durbin-Watson tests applied to the residuals of the fitted models

Model	Breusch-Pagan	Durbin-Watson	Shapiro-Wilk
Gompertz	0.2906	0.0000*	0.1401
von Bertalanffy	0.2564	0.0000*	0.3381
Brody	0.3851	0.0040*	0.2608
Chapman-Richards	0.5664	0.0000*	0.5038
Schöngart	0.4599	0.0020*	0.4133

* significant at 1% probability

Given the dependence of residuals, the fit was made by incorporating the first-order autoregressive parameter into the models, as according to Ribeiro *et al*. (2018b), in the presence of autocorrelated errors, it is important to model this autocorrelation and incorporate it into the model to ensure greater precision in the estimates. It is observed in Table 3 that, after the incorporation of the first-order autoregressive parameter to the models, all assumptions associated with the errors of the fitted models were validated (p-value > 0.01). Similar results were reported by Carneiro *et al.* (2014), Muniz *et al.* (2017), Jane *et al.* (2020) and Silva *et al.* (2020), who observed autocorrelated errors in the fit of nonlinear models applied to growth data in Tabapuan cattle, cocoa, sugarcane varieties and blackberry, respectively.

Table 3 - P-value for the Breusch-Pagan, Durbin-Watson and Shapiro-Wilk tests applied to the residuals of the models fit with the addition of the first-order autoregressive parameter

Model	Breusch-Pagan	Durbin-Watson	Shapiro-Wilk
Gompertz com AR(1)	0.2941	0.7340	0.0411
von Bertalanffy com AR(1)	0.4063	0.1680	0.8590
Brody com AR(1)	0.5124	0.0720	0.5270
Chapman-Richards com AR(1)	0.7929	0.2940	0.1814
Schöngart com AR(1)	0.5669	0.2940	0.4441

Figures 1 and 2 show the graphic distribution of residuals from the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models fit to the average height of *Eucalyptus* hybrid clones with the incorporation of the first-order autoregressive parameter (AR1).

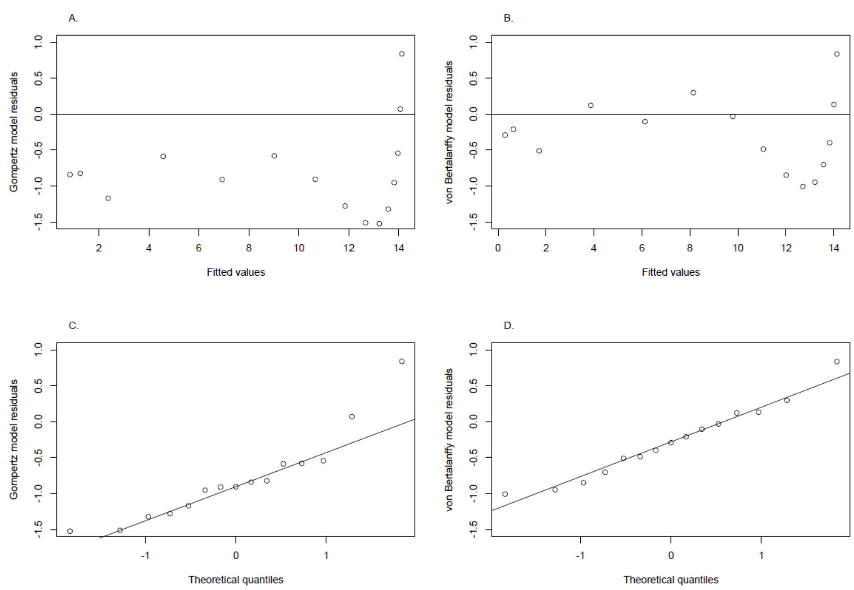

Figure 1 - Graphic distribution of residuals for average height of *Eucalyptus* hybrid clones, where (A) and (B) represent the fitted values in relation to the residuals and (C) and (D) represent the residual values in relation to the theoretical quantiles for the Gompertz and von Bertalanffy models with AR1.

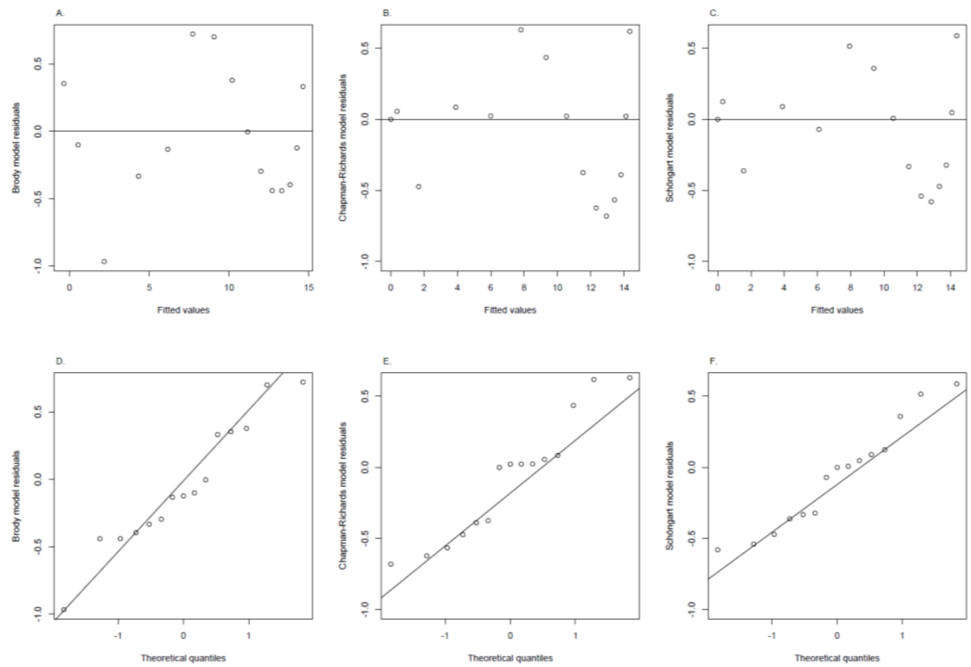

Figure 2 - Graphic distribution of residuals for average height of *Eucalyptus* hybrid clones, where (A), (B) and (C) represent the fitted values in relation to residuals and (D), (E) and (F) represent the residual values in relation to the theoretical quantiles for the Brody, Chapman-Richards and Schöngar models with AR1.

Table 4 lists the estimates for the parameters and their respective 95% confidence intervals based on the fit of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models for the average heights of *Eucalyptus* hybrid clones considering the structure of first-order autoregressive errors.

Table 4 - Estimates for the parameters of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models with first-order autoregressive error structure adjusted to average heights (m) of *Eucalyptus* hybrid clones and their respective 95% confidence intervals

Parameters	Gompertz			von Bertalanffy		
	LI	estimate	LS	LI	estimate	LS
α	11.9488	14.2321	16.5155	12.9466	14.4250	15.9033
β	9.0052	13.6715	18.3378	9.4862	13.0759	16.6656
κ	0.0522	0.0760	0.0999	0.0427	0.0597	0.0767
ϕ_1	0.5245	0.9085	0.9854	0.2392	0.7680	0.9455
Parameters	**Brody**			**Chapman-Richards**		
	LI	estimate	LS	LI	estimate	LS
α	14.3774	16.8099	19.2424	13.4813	15.1690	16.8566
β	0.9613	1.0210	1.0808	0.8813	1.4725	2.0637
κ	0.0179	0.0265	0.0351	0.0239	0.0422	0.0606
ϕ_1	0.0028	0.5227	0.8202	0.0636	0.6061	0.8721

Parameters	Schöngart		
	LI	estimate	LS
α	14.3184	17.0222	19.7260
β	19.2504	26.2267	33.2030
κ	1.1489	1.5538	1.9587
ϕ_1	0.0078	0.5673	0.8563

LI: lower limit; LS: upper limit

With the results in Table 4, it can be said that the estimated parameters were not null, as the zero value is not contained in the different 95% confidence intervals. All models had the parameter α, which expresses the maximum height of eucalyptus plants at 78 months of age in the semiarid region, estimated by close values and confidence intervals with similar amplitudes, with estimates ranging from 14.23 m to 17.02 m, corroborating the results of Ferreira *et al*. (2017).

The differences observed for the β parameter estimates are associated with the shape of the fitted curves. For the Gompertz and von Bertalanffy models, this parameter characterizes the abscissa of the inflection point of the curves (FERNANDES *et al*., 2015). The results obtained indicate that, for the region under study, the growth of eucalyptus plants starts to be decelerated from the 13 months of age of the trees, as observed in Figure 3, with maximum growth rates of 0.27 m/month and 0.38 m/month for the Gompertz and von Bertalanffy models, respectively.

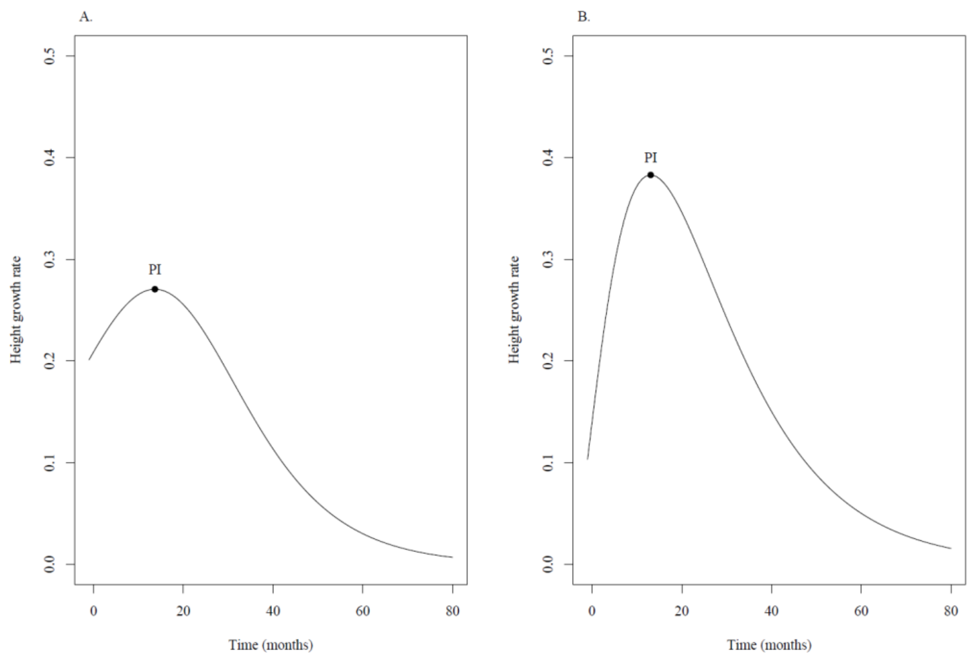

Figure 3 - Height growth rates of *Eucalyptus* hybrid clones and their inflection points (PI) obtained by the first derivative of the Gompertz (A) and von Bertalanffy (B) models.

To find the inflection point ordinate for the Gompertz and von Bertalanffy models, Mischan and Pinho (2014) suggest the use of the equations $\hat{y} = \frac{\hat{\alpha}}{e}$ and $\hat{y} = \frac{8}{27}\hat{\alpha}$, respectively, with e the Euler number ($e \approx 2.7182$). Thus, the inflection points (PI) have coordinates PI (13.67; 5.24) for the Gompertz model and PI (13.08; 4.27) for the von Bertalanffy model, indicating that the growth of trees in the study region starts to be decelerated at an average height of approximately 5 meters.

Table 5 lists the results of the criteria to assess the goodness of fit, suggesting that the Schöngart model best describes the data, as this model presented a higher value for R^2 and lower values for the AIC_C and DPR.

Table 5 - Criteria for assessing the goodness of fit of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models with first-order autocorrelated error structure

Models	R^2	AIC_C	DPR
Gompertz	0.9567	28.5171	1.0528
von Bertalanffy	0.9877	25.5872	0.6411
Brody	0.9916	26.3717	0.5039
Chapman-Richards	0.9932	22.5148	0.4727
Schöngart	0.9949	19.0864	0.4082

From the results found for the nonlinearity measures of Bates and Watts in Table 6, it can be observed that the fitted models presented adequacy of the linear approximations during the estimation process, as the intrinsic nonlinearity measures (c^{ι}) were not significant, obtaining values less than 0.3, a critical value for this study according to Ratkowsky (1983). Nevertheless, in the case of parametric nonlinearity (c^{θ}), the measurements were significant for all fitted models, as they all had values greater than 0.3, suggesting that their reparametrization is more adequate to describe the data, but as working with reparametrization is not the objective of this study, it is an option for future studies. It is important to emphasize that according to Silva (2021), the nonlinearity measures are important for the evaluation of the models, but they do not serve as a criterion for their selection.

Table 6 - Parametric (c^{θ}) and intrinsic (c^{ι}) nonlinearity measurements for the fit of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models with first-order autocorrelated error structure

Models	c^{ι}	c^{θ}
Gompertz	0.0732	0.3048
von Bertalanffy	0.0652	0.3163
Brody	0.0309	0.4990
Chapman-Richards	0.0622	0.6202
Schöngart	0.0648	0.6718

Figure 4 illustrates the fit of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models with first-order autoregressive error structure to average height data (m) over time (months). Visual analysis of the graphs indicates that the Schöngart model showed greater adherence to the observed values, corroborating the results obtained by the criteria used to assess the goodness of fit in Table 5.

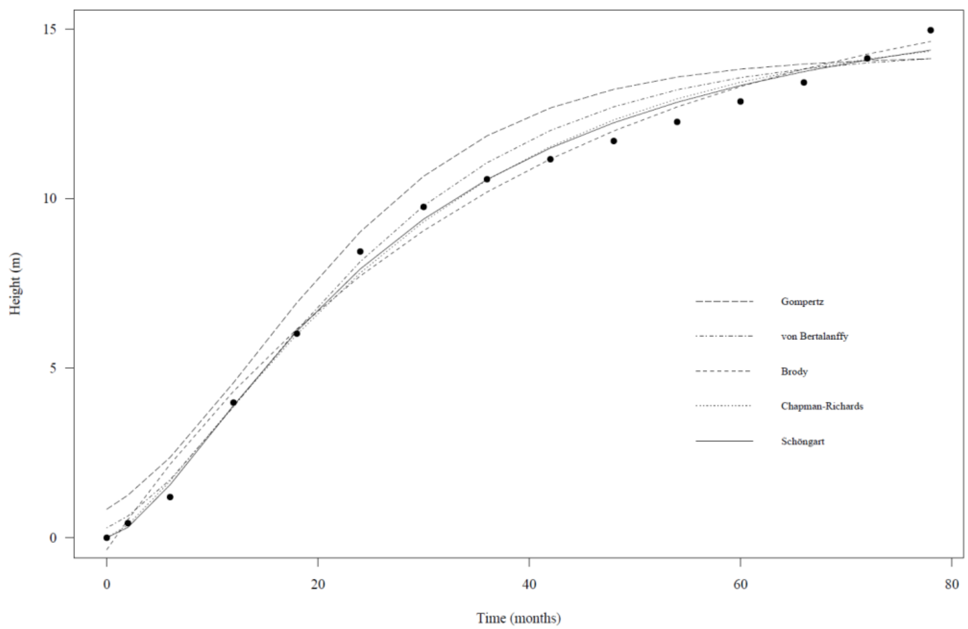

Figure 4 - Fit of the Gompertz, von Bertalanffy, Brody, Chapman-Richards and Schöngart models considering the first-order autoregressive error structure for the average height of *Eucalyptus* hybrid clones.

The Schöngart model, according to Table 4, obtained an estimate of approximately 17.02 meters for the maximum average height of *Eucalyptus* hybrid clone with 78 months, which is consistent with the results obtained by Ferreira *et al*. (2017) who found an approximate average value of 16.7 m and 19.7 m for the height of the clones of *Eucalyptus pellita* and *Eucalyptus urophylla*, respectively, in a period of 80 months.

The Chapman-Richards model has been widely applied in studies in the forestry area to describe the growth of several tree species, as can be seen in Lima Filho *et al.* (2012), Coutinho *et al.* (2017), Figueiredo Filho *et al.* (2017) and Silva *et al.* (2017), on the other hand, the Schöngart model is little used in the literature, with some studies analyzing the description of the growth of species in the Brazilian Amazon, such as those developed by Schöngart (2008) and Andrade *et al*. (2019). As this model proved to be the best fit for the description of the growth of *Eucalyptus* hybrid clones, the continuity of the study of this model can be of great value for the analysis of the growth of other tree species and other planting regions.

Conclusions

The Brody, Chapman-Richards, Gompertz, Schöngart and von Bertalanffy models with an autoregressive residual structure fit adequately in the description of growth in height data of *Eucalyptus* hybrid clones.

The residual dependence observed in all models was modeled by a first-order autoregressive structure, with estimated correlations above 0.5.

The Schöngart model was the one that best described the growth in height of *Eucalyptus* hybrid clones, and is the recommended model.

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- *RESUMO: O Brasil se destaca mundialmente pelo plantio de florestas homogêneas, principalmente de pinus e eucalipto. A produção florestal tem grande importância para economia do país, sendo também uma referência em sustentabilidade, competitividade e inovação. Dos 10 milhões de hectares de árvores plantadas, 76,3% dessa área é composta pelo gênero Eucalyptus, o que torna o Brasil um dos maiores produtores do mundo desse gênero. O estudo da trajetória de crescimento de árvores desse gênero pode ser um grande aliado no melhoramento dos planos de manejo atualmente utilizados. Neste sentido, o objetivo deste estudo foi comparar o desempenho dos modelos não lineares Gompertz, von Bertalanffy, Brody, Chapman-Richards e Schöngart, os quais foram ajustados através do software R considerando a estrutura de erros autorregressivos de primeira ordem (AR1), aplicados aos dados de altura média, em metros, em relação ao tempo, em meses, totalizando 15 observações obtidas durante seis anos e meio. Utilizou-se as medidas de não linearidade para verificar a adequabilidade das aproximações lineares dos modelos e como critérios para seleção do modelo o R2, AICC e DPR, sendo o modelo Schöngart (AR1) o que melhor se ajustou aos dados.*
- *PALAVRAS-CHAVE:* Curva de crescimento; Erros autorregressivos; Regressão; Schöngart.

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