Three- and four-point functions in *CPT*-even Lorentz-violating scalar QED

B. Altschul,[1,*] L. C. T. Brito,[2,†] J. C. C. Felipe,[3,‡] S. Karki,[1,§] A. C. Lehum,[4,‖] and A. Yu. Petrov[5,¶]

[1]*Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208, USA*
[2]*Departamento de Física, Instituto de Ciências Naturais Universidade Federal de Lavras, Caixa Postal 3037, 37200-900, Lavras, Minas Gerais, Brazil*
[3]*Instituto de Engenharia, Ciência e Tecnologia, Universidade Federal dos Vales do Jequitinhonha e Mucuri, Avenida Um 4050, 39447-790, Cidade Universitária, Janaúba, Minas Gerais, Brazil*
[4]*Faculdade de Física, Universidade Federal do Pará, 66075-110, Belém, Pará, Brazil*
[5]*Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brazil*

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The renormalization of quantum field theories usually assumes Lorentz and gauge symmetries, besides the general restrictions imposed by unitarity and causality. However, the set of renormalizable theories can be enlarged by relaxing some of these assumptions. In this work, we consider the particular case of a *CPT*-preserving but Lorentz-breaking extension of scalar QED. For this theory, we calculate the one-loop radiative corrections to the three- and four-point scalar-vector vertex functions, at the lowest order in the Lorentz-violation parameters, and we explicitly verify that the resulting low-energy effective action is compatible with the usual gauge invariance requirements. With these results, we complete the one-loop renormalization of the model at the leading order in the Lorentz-violating parameters.

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I. INTRODUCTION

Studies of Lorentz symmetry breaking represent an important line of research within the quantum field theory. The modern starting point of this line has been given by the foundational papers, Refs. [1,2], in which the Lorentz-violating (LV) standard model extension (SME) was formulated. In Ref. [2], the complete action of the minimal SME, which is an effective field theory that includes gauge, spinor, and scalar sectors, is written down, and some of the first examples of perturbative calculations are presented. The SME Lagrange density includes vector- and tensor-valued objects constructed out of existing quantum fields, which are contracted with background objects that represent favored spacetime direction structures. However, since the development of the theory, radiative corrections to the tree-level SME have been considered mostly in the context of spinorial quantum electrodynamics (QED) and its non-Abelian generalizations. (The results for lower-order radiative corrections in the minimal spinorial LV QED can be found in Ref. [3]; for a general review on radiative corrections in spinorial LV QED, see also Ref. [4] and references therein.)

There have been relatively few papers treating perturbative aspects of Lorentz violation in scalar field theories, including LV scalar QED. Among the key papers in the development of the scalar sector of the SME, we point especially to Ref. [5], which examines the modifications to the LV Abelian Higgs model; Ref. [6], in which the Higgs mechanism in LV scalar QED is further studied, with the additional inclusion of a *CPT*-violating Carroll-Field-Jackiw (CFJ) term and an analysis of the one-loop corections; Ref. [7], which looks at tree-level corrections to fermion scattering in LV Yukawa theory; Ref. [8], in which one-loop corrections in a scalar field theory with additional higher-derivative Myers-Pospelov-like LV terms are evaluated; Ref. [9], in which perturbative calculations in a simplified extension of the scalar QED sector of the SME are performed; and Ref. [10], in which one-loop effective potentials for various LV scalar field theories are calculated.

The study of field theories with broken Lorentz symmetry is not merely a subject of abstract theoretical interest. The real possibility that there could be tiny deviations from

*altschul@mailbox.sc.edu
†lcbrito@ufla.br
‡jean.cfelipe@ufvjm.edu.br
§karkis@email.sc.edu
‖lehum@ufpa.br
¶petrov@fisica.ufpb.br

perfect Lorentz invariance has been examined experimentally, in a wide variety of physical systems and using a remarkable array of different experimental techniques [11]. Active areas of experimental interest include measurements with gravitational waves [12–14], cosmic rays [15,16], neutrinos, and gamma-ray burst photons [17]. In the context of multimessenger astronomy, for example, we may have exciting new possibilities for testing and discriminating different models with Lorentz violation in the near future [18]. Because of the interest in these experimental tests, having a complete and accurate understanding of the relevant sectors of the SME is quite important, and a key part of understanding the theory is understanding the consequences of quantum corrections. Calculations of radiative corrections can play important roles in setting strong, reliable limits on LV background tensors or in interpreting evidence of physical Lorentz violation if it is ever uncovered.

In this paper, we will be looking at the scalar QED sector of the SME, including LV modifications not only in the scalar sector as has been done in Ref. [9] but also in the gauge sector as well. In both sectors, the LV terms are CPT even, having the standard SME forms $c^{\mu\nu}(D_\mu\phi)^*D_\nu\phi$ and $\frac{1}{4}\kappa^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ [2]. It is interesting to note that, if the constant tensors $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$ are characterized by only a single constant background vector u_μ, we recover an aetherlike theory [19]. This will be a continuation of the previous work [20], in which the one-loop corrections to the gauge and scalar two-point functions were explicitly evaluated. However, here, one of our main aims will be to check the gauge invariance of the quantum corrections in the scalar sector.

The structure of the paper is as follows. In Sec. II, we formulate the CPT-even LV scalar QED and write down the free propagators. In Sec. III, we calculate the one-loop corrections to the three-point correlation functions, and in Sec. IV, we give the four-point functions. In Sec. V, the final results are collected, and in Sec. VI, we look at renormalization group (RG) issues and explicitly evaluate the β-functions for the LV parameters. Finally, our results are summarized in Sec. VII.

II. MODEL

Our departure point will be the LV but CPT-even scalar QED Lagrange density (cf. Ref. [20]),

$$\mathcal{L} = (\eta^{\mu\nu} + c^{\mu\nu})(D_\mu\phi)^*(D_\nu\phi) - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}\kappa^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}, \tag{1}$$

where $D_\mu\phi = \partial_\mu\phi + ie\phi A_\mu$ is the covariant derivative, the metric signature is $\eta^{\mu\nu} = (1,-1,-1,-1)$, $m^2 > 0$ is a mass-squared parameter, and ϕ is the charged scalar field. Here, in contrast to Ref. [9], Lorentz symmetry breaking terms appear in both the scalar and gauge sectors—via the constant tensors $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$. As usual, we take both tensors to be traceless (for $\kappa^{\mu\nu\rho\sigma}$, by this we mean $\kappa^{\mu\nu}{}_{\mu\nu} = 0$), $c^{\mu\nu}$ to be symmetric, and $\kappa^{\mu\nu\rho\sigma}$ to display the same symmetry as the Riemann curvature tensor. Since $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$ will generally mix under radiative corrections, we expect that if one of them is present, they generally both must appear if the theory is to be strictly renormalizable. Note that we would also need a term $(\phi^*\phi)^2$ in the Lagrange density (1) to guarantee the full renormalizability of the model. However, we are omitting this term here because quantum corrections to it will not be affected by the Lorentz violation; this is a consequence of the tracelessness of the LV tensors.

The propagators of the theory, in momentum space, have the forms

$$G(k) = \langle\phi(k)\phi^*(-k)\rangle = \frac{i}{k^2 - m^2 + i\epsilon} - \frac{ic^{\mu\nu}k_\mu k_\nu}{(k^2 - m^2 + i\epsilon)^2} \tag{2}$$

$$\Delta^{\mu\nu}(k) = \langle A^\mu(k)A^\nu(-k)\rangle = -\frac{i\eta^{\mu\nu}}{k^2 + i\epsilon} + \frac{2i\kappa^{\mu\rho\nu\sigma}k_\rho k_\sigma}{(k^2 + i\epsilon)^2}. \tag{3}$$

For convenience, we have employed the Feynman gauge—adding the usual Lorentz-invariant gauge-fixing term $-\frac{1}{2}(\partial^\mu A_\mu)^2$ to (1). We then expanded the gauge propagator up to the first order in the LV parameters $c^{\mu\nu}$ and $\kappa^{\mu\rho\nu\sigma}$. As in Ref. [20] and as is commonplace in the literature, we are only taking into account the leading-order contributions from the LV background tensors. This is justified by the physical observation that Lorentz violation, if it exists, is known to be a small effect, so it makes sense to treat it as a perturbation on top of the well-understood Lorentz-invariant theory.

Both of the CPT-even terms with $c^{\mu\nu}$ and $\kappa^{\mu\rho\nu\sigma}$ were introduced originally in Ref. [2]. Because the coefficients $c^{\mu\nu}$ and $\kappa^{\mu\rho\nu\sigma}$ are dimensionless, it is expected based on power counting that the modified, LV scalar QED theory will continue to be renormalilzable. The tensor $c^{\mu\nu}$ of the present paper should not be confused with the one normally introduced in the fermionic sector of SME (although the two affect the dispersion relations for the scalar and fermion fields in homologous ways). What we are calling $c^{\mu\nu}$ was the $k_{\phi\phi}^{\mu\nu}$ tensor of Ref. [2], which can, most generally, possess a symmetric real part and an antisymmetric imaginary part. However, since we are considering here only a real $c^{\mu\nu}$, it must necessarily be symmetric.

Along with the modified propagators coming from the bilinear part of the Lagrange density (1), there are interactions, which are also modified by the presence of the $c^{\mu\nu}$ term. The vertices arise out of the presence of the covariant derivative $D_\mu\phi$ (which involves A_μ) in the action. It is a consequence of gauge invariance that same quantity $c^{\mu\nu}$ must appear in both the free scalar propagator and the three- and four-field gauge-scalar vertices. In the next two sections, we will be evaluating quantum correction involving the tree-level vertices

$$V_3 = i(\eta^{\mu\nu} + c^{\mu\nu})A_\mu(\phi\partial_\nu\phi^* - \phi^*\partial_\nu\phi) \tag{4}$$

and

$$V_4 = (\eta^{\mu\nu} + c^{\mu\nu})A_\mu A_\nu \phi\phi^*. \tag{5}$$

(We shall generally drop the coupling constant e, although it will be restored at the end of our analysis.) In order to evaluate all the relevant Green's functions, we shall use an adapted version of a set of *Mathematica* packages [21–23].

III. RESULTS FOR THE THREE-POINT FUNCTION

Now, we turn to the Feynman diagrams contributing to the three-point vertex function $\langle A\phi\phi^*\rangle$, which gives the quantum corrections to the vertex (4). All internal propagators now are "dressed," so that they depending on the LV parameters, given by (2) and (3). We obtain the Feynman rules for the vertices as usual from (4) and (5). The corresponding graphs are depicted in Fig. 1.

The contributions from the diagrams numbered 1–7 have the respective forms given by Eqs. (6)–(12) (with $p_3 = -p_1 - p_2$ from the conservation of the total incoming external momentum)

$$I_1 = i\phi(p_1)\phi^*(p_2)A^\mu(p_3)c^{\nu\rho}\int\frac{d^4k}{(2\pi^4)}G(k)G(k+p_3)\Delta_{\nu\sigma}(k-p_1)(2k+p_3)_\mu(k+p_1)_\rho(k+p_3-p_2)^\sigma \tag{6}$$

$$I_2 = i\phi(p_1)\phi^*(p_2)A^\mu(p_3)c^{\nu\rho}\int\frac{d^4k}{(2\pi^4)}G(k)G(k+p_3)\Delta_{\sigma\nu}(k-p_1)(2k+p_3)_\mu(k+p_1)^\sigma(k+p_3-p_2)_\rho \tag{7}$$

$$I_3 = i\phi(p_1)\phi^*(p_2)A^\mu(p_3)c_\mu{}^\nu\int\frac{d^4k}{(2\pi^4)}G(k)G(k+p_3)\Delta^{\rho\sigma}(k-p_1)(2k+p_3)_\nu(k+p_1)_\rho(k+p_3-p_2)_\sigma \tag{8}$$

$$I_4 = i\phi(p_1)\phi^*(p_2)A^\mu(p_3)\int\frac{d^4k}{(2\pi^4)}G(k)G(k+p_3)\Delta^{\rho\sigma}(k-p_1)(2k+p_3)_\mu(k+p_1)_\rho(k+p_3-p_2)_\sigma, \tag{9}$$

for the four diagrams with only three-field vertices internally and, for the last three diagrams (each of which includes a four-field vertex),

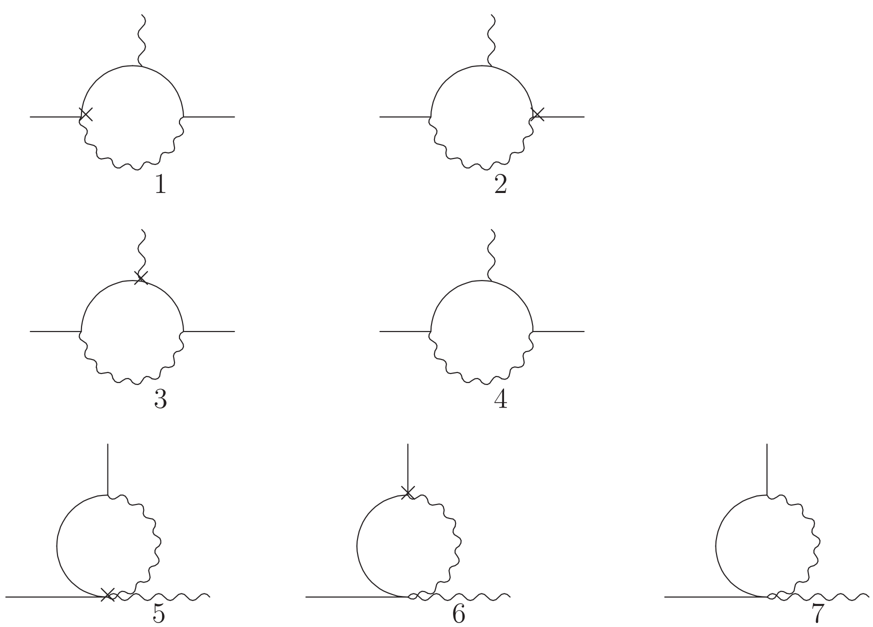

FIG. 1. Contributions to three-point gauge-scalar vertex. The wavy and solid lines represent the photon and scalar propagators, respectively, and the crosses indicate insertions of LV $c^{\mu\nu}$ parameters at the vertices. The usual Lorentz-invariant contributions come from portions of diagrams 4 and 7.

$$I_5 = 2i[\phi(p_1)\phi^*(p_2) - \phi^*(p_1)\phi(p_2)]A^\mu(p_3)c_\mu{}^\nu \int \frac{d^4k}{(2\pi^4)}\Delta_{\nu\rho}(k+p_2)(k-p_2)^\rho G(k) \tag{10}$$

$$I_6 = 2i[\phi(p_1)\phi^*(p_2) - \phi^*(p_1)\phi(p_2)]A^\mu(p_3)c^{\nu\rho} \int \frac{d^4k}{(2\pi^4)}\Delta_{\mu\nu}(k+p_2)(k-p_2)_\rho G(k) \tag{11}$$

$$I_7 = 2i[\phi(p_1)\phi^*(p_2) - \phi^*(p_1)\phi(p_2)]A^\mu(p_3) \int \frac{d^4k}{(2\pi^4)}\Delta_{\mu\nu}(k+p_2)(k-p_2)^\nu G(k). \tag{12}$$

The only diagrams that do not involve LV tensors at the vertices are I_4 and I_7. So, the Lorentz-violating contributions for these diagrams arise solely from the Lorentz-violating dressing of the propagators (2) and (3). In contrast, in the diagrams I_1–I_3 and I_5–I_6, we need only to keep Lorentz-invariant parts of the propagators, since the vertices ensure that the diagrams already depend on $c^{\mu\nu}$.

In order to calculate the vertex contributions I_1 through I_7, we may use the Implicit Regularization (IR) framework to isolate the divergent parts of amplitudes. (See Refs. [24–26] for general reviews of the method.) Here, we are interested only in the divergences of the radiative corrections, and thus we shall express the results in terms of the single logarithmically divergent integral

$$I_{\log}(m^2) = \int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)^2}. \tag{13}$$

The Λ annotation indicates that the integral is regularized in some manner compatible with the gauge symmetry. More precisely, this assumption is equivalent to adopting $\alpha_i = 0$ ($i = 1,\, 2$) in the general relations

$$\int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-m^2)^3} = \frac{\eta_{\mu\nu}}{4}[I_{\log}(m^2) + \alpha_1] \tag{14}$$

$$\int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\rho k_\sigma}{(k^2-m^2)^4} = \frac{\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}}{24} \times [I_{\log}(m^2) + \alpha_2]. \tag{15}$$

The IR method may also be applied to the naive quadratic divergences in scalar QED theories, replacing those formally divergent integrals with alternative expressions, in such a way that complete cancellation of all potential quadratic divergences is assured and transversality of the gauge theory is maintained. This entails setting the analogous α coefficients in the corresponding formal expressions for the integrals with power-counting quadratic divergences to be equal to zero.

Evaluating the integrals (6)–(12) is fairly tedious. After a lengthy calculation, we find that

$$I_1 = I_2 = \left[-\frac{2i}{3}c^{\mu\nu}(p_1-p_2)_\nu\right]I_{\log}(m^2)\phi(p_1)\phi^*(p_2)A_\mu(p_3) \tag{16}$$

$$I_3 = [-ic^{\mu\nu}(p_1-p_2)_\nu]I_{\log}(m^2)\phi(p_1)\phi^*(p_2)A_\mu(p_3) \tag{17}$$

$$I_4 = \left[\frac{2i}{3}c^{\mu\nu}(p_1-p_2)_\nu\right]I_{\log}(m^2)\phi(p_1)\phi^*(p_2)A_\mu(p_3) \tag{18}$$

$$I_5 = I_6 = [3ic^{\mu\nu}(p_1-p_2)_\nu]I_{\log}(m^2)\phi(p_1)\phi^*(p_2)A_\mu(p_3) \tag{19}$$

$$I_7 = \left[-i\left(\frac{1}{3}c^{\mu\nu} + \kappa^{\mu\rho\nu}{}_\rho\right)(p_1-p_2)_\nu\right] \times I_{\log}(m^2)\phi(p_1)\phi^*(p_2)A_\mu(p_3). \tag{20}$$

Each of these expressions has the structure of the momentum transfer at the vertex $p_1 - p_2$, contracted with a two-index symmetric tensor constructed out of the Lorentz-violating backgrounds. The terms in which the tensors that appear are simply multiples of $c^{\mu\nu}$ will contribute to the self-renormalization of the $c^{\mu\nu}$ tensor. However, the presence of $\kappa^{\mu\rho\nu}{}_\rho$ in (20) demonstrates explicitly that there is renormalization mixing between the background tensors from the gauge and scalar sector. On the other hand, it is interesting to note that the formula for I_4 does not depend on $\kappa^{\mu\nu\rho\sigma}$, since this background tensor is antisymmetric under the exchanges of the first two and of the last two indices.

In parallel with the *Mathematica* calculations, we also verified these expressions by hand. The manual calculations set the explicit integral expression for each diagram and used dimensional regularization (DR) to evaluate them.

These results (16)–(20) will be employed as part of the determination of the full gauge-scalar contribution to the one-loop effective action in Sec. V. Note that the RG β-function for the charge Lorentz-violating vertex coefficient may be determined solely from the leading quantum correction to the three-field vertex amplitude, in conjunction with the one-loop gauge and scalar self-energies. However, in order to confirm that the renormalization-improved theory remains gauge invariant, we also need to calculate the lowest-order radiative corrections to the four-field vertex.

IV. RESULTS FOR THE FOUR-POINT FUNCTION

So, the next step in our analysis will be calculating the radiative contributions to the four-point function $\langle AA\phi\phi^*\rangle$, which corrects the vertex (5). Note that the tree-level version of the vertex does not contain any derivatives.

In this case, the Feynman diagrams are those shown in Fig. 2. The analytical expressions for the potentially divergent parts of diagrams A–D are

$$I_A = -2\phi(p_1)\phi^*(p_2)A^\mu(p_3)A^\nu(p_4)(\eta_{\mu\rho}+c_{\mu\rho})(\eta_{\nu\sigma}+c_{\nu\sigma}) \times \int \frac{d^4k}{(2\pi)^4}\Delta^{\rho\sigma}(k)G(k) \tag{21}$$

$$I_B = 2\phi(p_1)\phi^*(p_2)A^\mu(p_3)A^\nu(p_4) \times (\eta_{\mu\rho}+c_{\mu\rho})(\eta_{\nu\sigma}+c_{\nu\sigma})(\eta_{\kappa\lambda}+c_{\kappa\lambda})(\eta_{\tau\varphi}+c_{\tau\varphi}) \times \int \frac{d^4k}{(2\pi)^4}[G(k)]^3\Delta^{\kappa\tau}(k)k^\rho k^\sigma k^\lambda k^\varphi \tag{22}$$

$$I_C = -2\phi(p_1)\phi^*(p_2)A^\mu(p_3)A^\nu(p_4) \times (\eta_{\mu\nu}+c_{\mu\nu})(\eta_{\rho\sigma}+c_{\rho\sigma})(\eta_{\kappa\lambda}+c_{\kappa\lambda}) \times \int \frac{d^4k}{(2\pi)^4}[G(k)]^2\Delta^{\rho\kappa}(k)k^\sigma k^\lambda \tag{23}$$

$$I_D = 2\phi(p_1)\phi^*(p_2)A^\mu(p_3)A^\nu(p_4) \times (\eta_{\mu\rho}+c_{\mu\rho})(\eta_{\nu\sigma}+c_{\nu\sigma})(\eta_{\kappa\lambda}+c_{\kappa\lambda}) \times \int \frac{d^4k}{(2\pi)^4}[G(k)]^2\Delta^{\lambda\sigma}(k)k^\rho k^\kappa. \tag{24}$$

Note that in obtaining these expressions, we have neglected all external momenta. They may be freely set to zero in the determination of the infinite renormalization of (5); this is related to the fact that the vertex V_4 is momentum independent. The naive degree of divergence of the diagrams shown in Fig. 2 is zero, so any appearance of a factor of an external leg momentum in the numerator of one of the integrals involved would render the integral involved finite —and thus negligible, since we are only considering the effects of the formally divergent radiative corrections.

These contributions must be expanded up to first orders in SME parameters. That is, we must again use the propagators expanded up to the first order in $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$. Proceeding in the same fashion as for the three-point functions, we have found the following results for the four-point vertex corrections:

$$I_A = (4c^{\mu\nu}+\kappa^{\mu\rho\nu}{}_\rho)I_{\log}(m^2) \tag{25}$$

$$I_B = -\frac{5}{6}c^{\mu\nu}I_{\log}(m^2) \tag{26}$$

$$I_C = 2c^{\mu\nu}I_{\log}(m^2) \tag{27}$$

$$I_D = -\frac{7}{6}c^{\mu\nu}I_{\log}(m^2). \tag{28}$$

(Note that the contributions from diagrams B–D actually sum to zero, leaving I_A as the sole contributor to the renormalization of the four-field vertex. However, while this observation seems potentially suggestive, it is actually specific to the Feynman gauge and does not hold more generally.) With these expressions, we have all we need to verify the gauge invariance of the model at the one-loop level.

V. FINAL RESULTS FOR THE LOW-ENERGY EFFECTIVE ACTION

In this section, we shall put together the results for the theory's two-point ($\langle\phi\phi^*\rangle$), three-point ($\langle A\phi\phi^*\rangle$), and four-point ($\langle AA\phi\phi^*\rangle$) correlation functions. This will enable us to perform an explicit verification of the gauge invariance of the renormalized effective action, at first order in the loop expansion and likewise first order in the Lorentz-violating parameters $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$. The expression for the two-point function was previously derived in Ref. [20], whereas the results for the three- and four-point functions have been calculated in the two preceding sections; the ultimate contributions from the diagrams in Figs. 1 and 2 are given by (16)–(20) and (25)–(28).

In order to maintain gauge invariance (and thus renormalizability and unitarity) at the perturbative order that we are interested in, the infinite parts of $\langle\phi\phi^*\rangle$, $\langle A\phi\phi^*\rangle$, and $\langle AA\phi\phi^*\rangle$ need to satisfy certain relations. The radiatively generated contributions to the effective action must, when taken together, have the same structure as the covariant

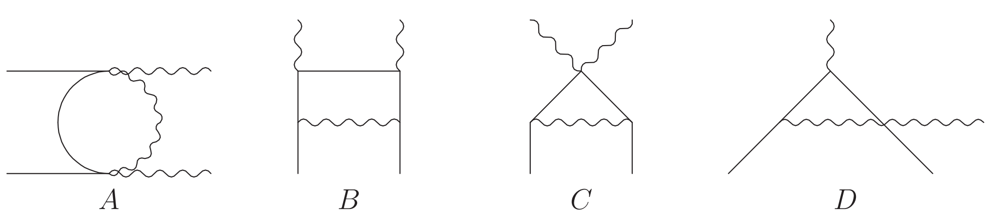

FIG. 2. Contributions to four-point gauge-scalar functions.

derivative term in the original action. The net contributions must assemble to form an expression of the form $k^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi)$, with some (logarithmically divergent) constant tensor $k^{\mu\nu}$. In momentum space, this kind of term takes the form

$$k^{\mu\nu}[-p_{1\mu}p_{2\nu} - i(p_1 - p_2)_\mu A_\nu(p_3) + A_\mu(p_3)A_\nu(p_4)]\phi(p_1)\phi^*(p_2). \tag{29}$$

In fact, the sum of all diagrams in Figs. 1 and 2, as well as the scalar self-energy diagrams [20], takes precisely this form. The divergent part $\Gamma_{\rm div}$ of the resulting sum looks like

$$\Gamma_{\rm div} = (4c^{\mu\nu} + \kappa^{\mu\rho\nu}{}_\rho)I_{\log}(m^2)[-p_{1\mu}p_{2\nu} - i(p_1 - p_2)_\nu A_\mu(p_3) + A_\mu(p_3)A_\nu(p_4)]\phi(p_1)\phi^*(p_2), \tag{30}$$

which clearly matches (29).

VI. RENORMALIZATION GROUP FUNCTIONS

So far, we have derived the divergent part of the effective action for the model (1). In this section, we shall compute the RG functions associated with the theory, in particular the β-functions that describe the dependences of the SME terms on the interaction scale. For clarity, we shall take the $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$ tensors to have particularly simple forms, so that the RG scaling for each of them may be reduced to the behavior of a single scalar quantity. However, it would be a completely straightforward generalization to separate out the individual β-functions for the individual Lorentz components of the tensors.

The specific form we shall assume for the matter-sector SME coefficients is

$$c^{\mu\nu} = Q_c u^\mu u^\nu, \tag{31}$$

where u^μ is a fixed unit(like) dimensionless four-vector. A theory in which all the LV backgrounds depend on just a single such four-vector are (especially when the vector in question is purely timelike) often referred to as "aetherlike" LV theories. In the context of spontaneous breaking of Lorentz symmetry, models with just a single preferred four-vector background are "bumblebee" models [27]. The bumblebee framework involves a single dynamical four-vector field acquiring a vacuum expectation value, which sets the spacetime direction of u^μ. In principle, the field could be timelike, spacelike, or lightlike (depending, for example, on the structure of the potential responsible for spontaneous Lorentz symmetry breaking). However, the expression (31) is subtly defective if $u^2 \neq 0$, since in that case the trace $c^\mu{}_\mu$ is nonzero, contrary to standard conventions. It would be quite straightforward to rectify this problem by subtracting an additional diagonal tensor from (31). However, in the interest of maintaining maximal simplicity in our calculations, we shall instead assume that u^μ is simply lightlike—$u^2 = 0$ implying that $c^{\mu\nu}$ is traceless.

The Lorentz violation coefficient $\kappa^{\mu\nu\rho\sigma}$ in the pure electromagnetic sector will also be taken to depend on just the lightlike u^μ and an overall normalization constant. The specific form is

$$\kappa^{\mu\nu\rho\sigma} = Q_\kappa(u^\mu u^\rho\eta^{\nu\sigma} - u^\mu u^\sigma\eta^{\nu\rho} + u^\nu u^\sigma\eta^{\mu\rho} - u^\nu u^\rho\eta^{\mu\sigma}). \tag{32}$$

Like Q_c, Q_κ is dimensionless. When we need to denote the bare versions of quantities, we shall include an additional subscript 0, so that $Q_{\kappa 0}$ and Q_{c0} stand for the bare parameters which appear in bare version of the Lagrange density (1).

Although it is possible to calculate the RG functions using the IR formalism directly [28], in this section, we will express our results in the more commonly used language of DR. It is actually simple to carry expressions over from IR to DR. In IR, the RG scale μ is introduced via the identity

$$I_{\log}(m^2) - I_{\log}(\mu^2) = \frac{i}{16\pi^2}\ln\left(\frac{\mu^2}{m^2}\right). \tag{33}$$

To obtain the same results as in conventional DR, we may just set all the α_i parameters to zero in (14) and (15), use the identity (33) to write the Green's functions as a function of μ, and substitute for the $I_{\log}(\mu^2)$ defined in (13) the DR formula

$$\mu^{D-4}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-\mu^2)^2} = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon} + \ln 4\pi - \gamma\right) + \mathcal{O}(\epsilon). \tag{34}$$

Here, we defined as usual $\epsilon = \frac{D-4}{2}$, where D is the analytically continued dimension of spacetime. In what follows, we will be interested only in the singular term containing $1/\epsilon$ in (34).

We define the renormalized fields $\phi_0 = Z_2^{1/2}\phi$ and $A_0^\mu = Z_3^{1/2}A^\mu$ and rewrite the Lagrangian (1)—taken to depend on the bare fields—in terms of the renormalized fields (and at this stage, we also restore the previously omitted coupling constant e, which is equivalent to taking the bare charge to be $e_0 = 1$)

$$\mathcal{L} = Z_2(\partial^\mu\phi^*)(\partial_\mu\phi) - m^2 Z_m\phi^*\phi - \frac{1}{4}Z_3 F^{\mu\nu}F_{\mu\nu} + ieZ_1A^\mu(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) + e^2Z_4A^\mu A_\mu\phi^*\phi + u^\mu u^\nu[Q_cZ_5(\partial_\mu\phi^*)(\partial_\nu\phi) + Q_\kappa Z_6F^{\mu\alpha}F^\nu{}_\alpha] + u^\mu u^\nu[ieQ_cZ_7A_\nu(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) + e^2Q_cZ_8\phi^*\phi A_\mu A_\nu] + \mathcal{L}_{GF}. \tag{35}$$

Here, $\mathcal{L}_{GF}$ is the gauge-fixing term, and the relations among the renormalization constants are

$$m^2Z_m = \mu^{-2\epsilon}m_0^2Z_2 \tag{36}$$

$$eZ_1 = \mu^{-2\epsilon}Z_2Z_3^{1/2} \tag{37}$$

$$e^2Z_4 = \mu^{-2\epsilon}Z_2Z_3 \tag{38}$$

$$Q_cZ_5 = \mu^{-2\epsilon}Q_{c0}Z_2 \tag{39}$$

$$Q_\kappa Z_6 = \mu^{-2\epsilon}Q_{\kappa 0}Z_3 \tag{40}$$

$$eQ_cZ_7 = \mu^{-2\epsilon}Q_{c0}Z_2Z_3^{1/2} \tag{41}$$

$$e^2Q_cZ_8 = \mu^{-2\epsilon}Q_{c0}Z_2Z_3. \tag{42}$$

The definitions of the renormalization constants account for the fact that the bare Lagrangian effectively had $e_0 = 1$. Each of the renormalization constants Z_i may be expanded as a power series in the coupling constants and determined sequentially, order by order in the perturbative expansion—that is,

$$Z_i = 1 + Z_i^{(1)} + Z_i^{(2)} + \cdots. \tag{43}$$

We now need to evaluate the counterterms corresponding to the ultraviolet-divergent parts of the one-loop corrections to the scalar and photon self-energies (for Feynman diagrams calculated in Ref. [20]) and the vertex functions (given by Figs. 1 and 2 of this paper) in the scalar QED model. We start with the one-loop scalar self-energy. The insertion of the LV parameters at the first order is considered in Ref. [20]. The resulting expression is

$$\Sigma_1(p) = \frac{\lambda m^2 - e^2(m^2+2p^2)}{16\pi^2\epsilon} - \frac{e^2(4Q_c+Q_\kappa)(u\cdot p)^2}{16\pi^2\epsilon} + Z_2^{(1)}p^2 - m^2Z_m^{(1)} + Q_cZ_5^{(1)}(u\cdot p)^2, \tag{44}$$

from which it is possible to read off what the leading contributions to the renormalization constants must be,

$$Z_2^{(1)} = \frac{e^2}{8\pi^2\epsilon} \tag{45}$$

$$Z_m^{(1)} = \frac{(\lambda - e^2)}{16\pi^2\epsilon} \tag{46}$$

$$Z_5^{(1)} = \frac{e^2(4Q_c+Q_\kappa)}{16\pi^2Q_c\epsilon}. \tag{47}$$

Note that in (44) and (45), we have tacitly added back in the usual contribution from the four-scalar coupling λ. Due to the tracelessness of $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$, the renormalizations of the usual field strength (Z_2) and mass (Z_m) terms are not affected by the Lorentz violation.

The one-loop photon self-energy with the LV parameters insertions is similarly given by [20]

$$-i\Pi^{\mu\nu}(p) = -(p^2\eta^{\mu\nu} - p^\mu p^\nu)\left[\frac{e^2}{48\pi^2\epsilon} + Z_3^{(1)}\right] + \{(u\cdot p)[\eta^{\mu\nu}(p\cdot u) - u^\nu p^\mu] + u^\mu[u^\nu p^2 - (u\cdot p)p^\nu]\}\left[-\frac{e^2Q_c}{48\pi^2\epsilon} + \frac{Q_\kappa Z_6^{(1)}}{2}\right]. \tag{48}$$

The renormalization constants needed to subtract the divergences are

$$Z_3^{(1)} = -\frac{e^2}{48\pi^2\epsilon} \tag{49}$$

$$Z_6^{(1)} = \frac{e^2Q_c}{24\pi^2Q_\kappa\epsilon}, \tag{50}$$

$Z_3^{(1)}$ naturally being the usual Maxwell field strength renormalization constant, again because of the tracelessness of the SME background tensors.

To the older results for the two-point function counterterms, we now add the vertex correction terms, beginning with the ultraviolet-divergent part of the three-point vertex function. Using the definition (32) in the results of Sec. III, and adding the contributions (16)–(20), we obtain

$$-i\Gamma^{\mu} = e(p_2^{\mu} - p_1^{\mu})\left[\frac{e^2}{8\pi^2\epsilon} - Z_1^{(1)}\right] + eQ_c u^{\mu}[u\cdot(p_2 - p_1)] \times \left[\frac{e^2(4Q_c + Q_\kappa)}{16\pi^2 Q_c\epsilon} - Z_7^{(1)}\right]. \tag{51}$$

Imposing finiteness, we have

$$Z_1^{(1)} = \frac{e^2}{8\pi^2\epsilon} \tag{52}$$

$$Z_7^{(1)} = \frac{e^2(4Q_c + Q_\kappa)}{16\pi^2 Q_c\epsilon}. \tag{53}$$

The expressions derived so far contain enough information to determine the β-functions for Q_c and Q_κ. However, to verify the gauge invariance of the renormalized theory, we also need to look at the four-field vertex corrections (although some of the necessary relations may already be checked at this stage). Specifically, we need to look at the radiative corrections to the gauge-scalar four-point function. [As noted above before, we are not interested in the quantum corrections to the four-field $(\phi^*\phi)^2$ vertex because these corrections will not be affected by the Lorentz violation if $u^2 = 0$.] Using (31) and (32) and adding the results (25)–(28), we find

$$\Gamma^{\mu\nu} = -\frac{e^4\eta^{\mu\nu}}{4\pi^2\epsilon} - \frac{2e^4(4Q_c + Q_\kappa)u^{\mu}u^{\nu}}{16\pi^2\epsilon} + 2e^2 Z_4^{(1)}\eta^{\mu\nu} + 2e^2 Q_c Z_8^{(1)} u^{\mu}u^{\nu}, \tag{54}$$

which is rendered finite if

$$Z_4^{(1)} = \frac{e^2}{8\pi^2\epsilon} \tag{55}$$

$$Z_8^{(1)} = \frac{e^2(4Q_c + Q_\kappa)}{16\pi^2 Q_c\epsilon}. \tag{56}$$

Since there are supposed to be, according to (36)–(42), only two underlying divergent renormalization factors in the theory, we can test the consistency of our calculations with gauge invariance—for example, by comparing (45) with (52) and (47) with (53) and (56). The consistency conditions are indeed satisfied, and, in particular, we have

$$Z_1^{(1)} = Z_2^{(1)} \tag{57}$$

$$Z_5^{(1)} = Z_7^{(1)} = Z_8^{(1)}, \tag{58}$$

which are the Ward identities for the theory. In particular, the identities (58) are directly related to the structure we found in (30) for the effective action, and thus they are an explicit manifestations of the gauge-invariant structure of our result.

Now, we are prepared to compute the RG functions of the model. Based on the relations defining the Z_i factors and their specific values obtained in this section, we find the following expressions for the β-functions:

$$\beta(e) = \mu\frac{de}{d\mu} = \frac{e^3}{48\pi^2} \tag{59}$$

$$\beta(\lambda) = \mu\frac{d\lambda}{d\mu} = \frac{5\lambda^2 - 12\lambda e^2 + 24e^4}{16\pi^2} \tag{60}$$

$$\beta(Q_c) = 3\beta(Q_\kappa) = \frac{e^2(2Q_c + Q_\kappa)}{8\pi^2}. \tag{61}$$

As expected, $\beta(e)$ and $\beta(\lambda)$ are the same as in standard scalar QED without LV terms.

It may also be interesting to notice some of the consequences of relaxing the assumption $u^2 = 0$ in the calculations of the RG functions. The anomalous dimensions $\gamma_i = \frac{1}{2}\frac{d\ln Z_i}{d\ln\mu}$ are given by

$$\gamma_2 = \frac{1}{2}\frac{d\ln Z_2}{d\ln\mu} = -\frac{e^2}{8\pi^2} + \frac{3e^2u^2(Q_c - Q_\kappa)}{64\pi^2} \tag{62}$$

$$\gamma_3 = \frac{1}{2}\frac{d\ln Z_3}{d\ln\mu} = \frac{e^2}{48\pi^2} - \frac{e^2u^2Q_c}{64\pi^2} \tag{63}$$

$$\gamma_m = \frac{1}{m}\frac{dm}{d\ln\mu} = -\frac{2e^2 - \lambda}{16\pi^2} + \frac{\lambda u^2 Q_c}{16\pi^2}, \tag{64}$$

corresponding to the fact that if $u^2 \neq 0$, the scalar field in the Lagrange density is no longer canonically normalized, whereas for $u^2 = 0$, these functions are equal to the usual ones, calculated without Lorentz violation. The consequences of relaxing the tracelessness property of $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$ are potentially more interesting for $\beta(e)$, since in that case we must add the term $-\frac{e^2u^2Q_c}{96\pi^2}$ to the result (60). For a timelike aetherlike vector with $u^2 = 1$, along with $Q_c > 0$, it appears that a nontrivial fixed point for $\beta(e)$ may arise out of the LV interactions, at $e_* = \frac{Q_c}{2}$. This is suggestive, and it contrasts sharply with the $u^2 = 0$ case we have mostly concentrated on—in which the Lorentz violation does not contribute to the RG running of the electric charge at one-loop order. In any case, we anticipate that our results may be useful for more detailed future studies of RG behavior in the SME gauge and scalar sectors.

VII. CONCLUSION

This work essentially completes the one-loop renormalization of the SME's scalar QED sector, to first order in the CPT-even LV terms in the scalar ($c^{\mu\nu}$) and gauge ($\kappa^{\mu\nu\rho\sigma}$) sectors. These results broaden our understanding of the perturbative structure of a frequently neglected corner of the SME. Specifically, we have calculated the three- and four-point gauge-scalar vertex corrections in this model. For both functions, we have confirmed that the results, when combined with previously calculated two-point functions, yield correctly proportionate contributions to the gauge-invariant structure $k^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi)$ in the effective Lagrange density.

The radiatively generated quantity $k^{\mu\nu}$ receives contributions proportional to both $c^{\mu\nu}$ and $\kappa^{\mu\rho\nu}{}_\rho$. As expected in a renormalizable theory with dimensionless couplings, both sets of radiative corrections are formally logarithmically divergent, so they must be regulated and renormalized. (While in theories with fermionic loops, some of the similar contributions may vanish due to their Dirac matrix structures forcing certain traces to be zero, such a cancellation mechanism clearly cannot work in purely bosonic theories.) Furthermore, using our calculations of the renormalization constants of the model, we were also able to calculate RG functions for the LV operators.

It is interesting to note that corrections to the scalar four-point function $\langle(\phi^*\phi)^2\rangle$ at first order in $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$ are actually zero, because the tensors are taken to be traceless. Moreover, if the background tensors obey a particular relation, namely $4c^{\mu\nu}+\kappa^{\mu\rho\nu}{}_\rho=0$, all the divergent contributions to $k^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi)$ cancel out. In this case, all the one-loop radiative corrections to the "covariantized" kinetic term in the scalar field Lagrange density are finite. This resembles the situation discussed in Ref. [29], in which the divergent contributions to an effective CFJ term also turned out to vanish if the SME parameters involved satisfied a special relation.

The natural continuations of this study could consist of, first, a more detailed evaluation of finite radiative corrections, which can contribute to cross sections and similar quantities in LV scalar QED; second, development of higher-order calculations, including both higher-loop Feynman diagrams and calculations at second and higher orders in $c^{\mu\nu}$ and $\kappa^{\mu\nu\rho\sigma}$; third, inclusion of the possibility of spontaneous gauge symmetry breaking; and fourth, extension of these results to LV theories of non-Abelian gauge fields coupled to scalar matter. We intend to undertake further studies in these directions in subsequent papers. In particular, the non-Abelian version of the theory represents one of the last remaining components of the minimal SME whose one-loop renormalization has not yet been completed, although some of the necessary calculations are fairly straightforward generalization of ones that have already been done. For example, to generalize the three-point scalar-vector diagrams shown in Fig. 1, it is only necessary to keep track of internal non-Abelian group generators at the vertices and to include diagrams in which the external vector line is attached to the internal gauge propagator with a three-gauge-boson vertex. Ultimately, all of these quantum correction calculations will enhance our understanding of possible Lorentz violation in scalar field dynamics, including elucidating possible experimental signatures of Higgs-sector Lorentz violation.

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