FORMATTING MODAL OF AN INTEGRATED URBAN REGION DESIGNED FOR THE PRE-HOSPITALIZATION ASSISTANCE TRANSPORT SYSTEM

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■ *ABSTRACTT: The mobile pre-hospitalization service for individuals is a specialized assistance service outside the hospital establishments. The intention of this type of service is to maximize the assistance aiming to preserve life. The purpose of this kind of service is to provide to the user – the person who suffered an accident in a certain region R, the shortest response time after the incident occurred. The means of transport for the removal of the victims to the hospital establishments is done by a service unit (U_s). In this way, the substantial increase in the value curve expected for the assistance throughout time is a very well known fact. For the pre-hospitalization service, a city can be treated as a single urban region of assistance or can be stratified into urban zones of service. For the first case, a single station or location serves as the screening headquarters – the locality is used as a distributor of the U_s and, in this case, the patient is removed to the closest hospital establishment, where the user of the system (accident victim) is located. The second case, in each assistance zone (ZA) is hospital establishment is allocated, such as, public and private establishments of the municipality or the state in accordance with the National Health System (NHS). In this way, the model to be approached for assistance in a region is the representation of the urban areas of the city and the urban zones of assistance (ZA) as a function of the area elements of the region that contain a hospital organization. In short, each one of the zones of assistance is defined to minimize the time response (TR) between the call to the U_s and the arrival of the service to the victim. This is the only intention to guarantee the accident victims' lives. So, we must look for statistical methods of analysis which may solve the problems of geographic location and the points in the network of assistance, measuring in a robust manner the distances between these points and assuring to situate them in an optimized method, under some transport restrictions, such as time duration and cost.*

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■ *KEYWORDSSS: Assistance zone; hospital system; integration network; location of the service posts; metrics, minimum mistance; rayleigh distribution; response time.*

1 Introduction

The mobile pre-hospital assistance is a service of individualized and specialized assistance outside the hospital establishments. The purpose of this type of service is to maximize the assistance aiming for the preservation of life. This kind of assistance has, as its objective, to provide the user – the individual who suffered an accident in a determined region R, in the shortest time response, after the incident happened; being imperative to provide adequate assistance to the victim. The means of transportation for removal of those who suffered an accident to the hospital establishments is done by a unit service (U_s).

The mortality profile has altered through the last decades, in Brazil, as well as worldwide. On the one hand, the improvement of sanitary conditions and medical progress have reduced deaths of various diseases; overabundance of automobiles, sedentary lives, longevity and urban violence, amongst other factors, created or accentuated medical urgencies deriving from traumas, mainly, motivated by traffic accidents. Being as it is, a substantial increase within the expected value curve of the assistance, throughout the period, is widely known.

Many of these deaths could have been prevented if the assistance to the victim had been during the first instance after the cause of the medical urgency, which depends on the outline of the geometry of the service zone and is determinant for their survival.

This moment depends basically on the number of the U_s units – service assistance and of the localization of the stations or service posts where these U_s posts are allocated. Obviously when a quantity of the U_s becomes available, the average time for an accident assistance in an emergency decreases substantially when the U_s service units have been favorably distributed in the region R, based on strictly scientific criteria.

The American regulation for this type of service establishes that 95% of the solicitations in a given region R of the urban area be attended in a maximum of 10 minutes. This time can be extended to 30 minutes when R is in a rural area (Ball and Lin, 1993). In the case of Brazil, specifications in the legislation determining the higher or lower limits for the time response do not exist.

To establish these limits, it is necessary to define a distribution for the U_s through statistical models that consider all of the variables inherent to the full service of the solicited U_s, for instance, time, accident rates, distances, number of beds in the hospital establishments, size of the zones of assistance and the user population through the assistance of that region.

It is known that the mobile pre-hospitalization service presents high degree of uncertainty in its operational characteristics and the higher the degree of uncertainty, higher the necessity for swift answers.

In Brazil, according to the study report "Economic Impact of the Traffic Accidents in the Brazilian Urban Agglomerations - IPEA (2004)", the costs associated to these loses come to represent an average of $0,4\%$ of the Gross Domestic Product (GPD) of the nation.

2 Zoning: Division of the urban into zones of assistance – ZAs

For the pre-hospitalization assistance, we can deal with a city as a single urban region of assistance or stratify it in urban assistance zones. For the first case, one single station or post serves as a selection central – the locality is used as an U_s distributor and, in this case, the patient (accident victim) is removed to the closest hospital establishment in that vicinity where the user of the system is. The second case, in each assistance zone (ZA) an established hospital is allocated, such as, a public and private hospital of the municipality and of the state in accordance with the National Health System (NHS).

In this way, the profile to be taken into consideration for the service in a region is the representation of all the urban areas of the city; the urban zones of assistance (ZA) function as elements in the areas of the regions and contain a hospital organization. Consequently, each one of the zones of assistance is defined for the minimization of the time response (TR) – time between the call to the U_s and its arrival to the patient. This is the single purpose to guarantee the victims' life.

One of the simplest forms for the definition of the ZAs is to format the zones into figures close to circles and squares, plotting the hospital establishment in the center of the figure achieving the shortest travel time. It is known through the Euclidian geometry, that among all the quadrilaterals of the same perimeter, the square is the plane figure that establishes the shortest distance expected between two points chosen randomly inside it.

Each hospital establishment has a station – a service post in the hospital service unit (U_s) or simply service posts whose purpose is of exclusively attending the calls inside corresponding assistance zones. Hence, this procedure produces a homogenization of the service rendered, as well as a better use of the people who work in the $U_s's$. This system where various employees are distributed in the zones ZAs propitiates the creation of a system of lines in each unit zone station (post) in order to attend the calls giving priority: "First in First out (FIFO)". For the sizing up of this system of lines we start from the principle that all the calls are classified as urgent, not taking into consideration those which are unnecessary or lost.

The pre-hospitalization transport system intended to attend exclusively in emergencies as a means of transportation is the service unit (U_s), which has two peculiarities exceptionally relevant: the first refers to the method of $U_s's$ lines and the second to the time of travel TV.

The use of the queueing theory has as main objective to reduce to zero the probability that the call will continue in a queue for a certain period of time very near to zero. In this sense, the theoretical basis for the queueing theory is to establish equations seeking to reduce the average time of waiting in line $\overline{W}$ considerably, that is to say, sizing the U_s in a manner making $\overline{W}$ sufficiently reduced in relation to the time of the trip. Being as it is, it becomes necessary to increase the idle time, and reduce the use factor ρ.

While doing this search – increasing the idle time by means of reduction use ρ to have $\overline{W} \equiv 0$ is to incur in greater costs mainly those attributed to capital and labor. Regarding the capital, depreciation is the uppermost weight. However, the increase of cost regarding labor is due to low use of the driver, especially its misuse related to the hours of idleness. This is without doubt the price to be paid because of reducing $\overline{W} \equiv 0$.

With regard to the time of the trip, the objective is to choose a geometric format for the zone of assistance ZA as to minimize $E(TV)$. Like the time of waiting in line $\overline{W} \equiv 0$, the time of the trip TV participates mostly in the time response equation $TR = W + TP + TV$, where W is the waiting time and TP is the time spent for an U_s service unit storage to depart, that is, the time between the occupation up to its departure from the place of storage.

Assuming that all the U_s that serve in pre-hospital service to a particular city are allocated in a single sorting station – one service post only – there is an effect, which is the not reduction of the TR because of the size/dimension of the area of the service attended, since even having the number of U_s altered, the rebate of the time response TR is minimum. In this way, it becomes unfeasible to serve the whole urban region as being one zone of assistance; a single station or sorting station for the distribution of the calls of all service units (U_s).

By abandoning the hypothesis of concentrating all the U_s in one service station for U_s dispatches, it sets out to distribute the U_s into ZAs. In such case, the travel time TV suffers modification due to the geometric characteristics of ZAs. Thus, it is necessary to build or acquire the models that already exist, that propitiate to establish a mathematical relation between the travel time TV with the geometrical characteristics of the ZAs. There are geometric models that estimate randomly TV, considering that the locality of the service station is known, as well as its population. The population considered is elucidated by a function uniformly distributed in every ZA where the service station is located. There are several studies that examine policies of dispatch of mobile pre-hospitalization assistance, like in Takeda et al. (2004, 2007) which show clearly, that the decentralization of service units can improve the efficiency of a pre-hospitalization mobile system through the substantial reduction in the average time response $E(TR)$ of the injured users. Ignall et al. (1982) simulated various policies of firefighter vehicles dispatched due to the motive/severity of the assistance and the number of vehicles that should be sent when an alarm occurs.

The conception of a model of lines is to guarantee that at least one U_s service unit be available for assistance when called; with this, has the condition to calculate the probability for this guarantee. The geometric model, the distribution of travel time is made according to this TV, guarantees, minimizing the expected value of time response TR, therefore verifying the compatibility of ZA. In this way, the search of efficiency maximization for the pre-hospital assistance service using U_s will be done according to the assertions explained by the following equations:

$$E(TR) = \quad E(TP) + E(TV) \leq TMR \quad \text{and} \tag{1}$$

$$\overline{W} \equiv 0, \tag{2}$$

where TMR represents the maximum specified time response for any ZA, according to the geometric shape that leads to a minimum $E(TR)$.

The Brazilian capitals are divided into administrative regions, which in the case of Recife – the capital of the Pernambuco state denominated the Administrative Policy Regions ($RPAs$) covering several districts, whose population density is constant. This constant density of around 65 inhabitants per hectare (quotient among the population of 1.422.905 inhabitants compactly in an area of 21.964 ha) shows that in all major Brazilian capitals, the number of administrative regions is always inferior to the number of hospitals.

The city of Recife has 89% of its land property built, which is distributed stable among the six $RPAs$, which present percentages ranging from 84% (west region) to 95% (central region). The southern region RPA6 has the highest population concentration, which corresponds to 24,9% of the entire city. The small urban area of Recife has 251 public health facilities considering those with NHS healthcare plus the private nonprofit healthcare ($IBGE$, 2005). Among the 251 hospital facilities, 41 (21 public plus 20 private which are associated to NHS) are hospital establishments that offer specialized care or hospitalization or with various specialties. The average number of hospitals with these characteristics by RPA is 6,12.

The Organic Law of Recife was supported in a participatory planning process, consolidated through the $RPAs$, based on the following territorial division: 94 districts and 6 $RPAs$, being: $RPA1$ - Center: 11 districts; $RPA2$ - North: 18 districts; $RPA3$ - Northwest: 29 districts; $RPA4$ - West: 12 districts; $RPA5$ - Southwest: 16 districts; $RPA6$ - South: 8 districts. The Brazilian Institute of Geography and Statistics ($IBGE$, 2005) in its demographic census utilizes this division to elaborate jointly with the city hall the maps and tables containing the resident population, number of domiciles, area, density of the regions and other statistics of great value for the Administrator Plans.

Table 1 illustrates the resident population, the number of private domiciles, area and density of the $RPAs$ of the city of Recife.

Based on the number of beds n_L of the hospital facility the established parameter (λ) and (μ) (Average rate of accidents of an user – entrance rate) and (average rate of operation, user without service – departure rate), respectively, for a given town, we aim to determine the maximum user population at that

Table 1 - Resident Population, Number of Private Households, Area and Density of the $RPAs$ – Recife/2005

Political Administrative Region	Resident Population (Abs)	(%)	Domic.	Area(ha)	Density (hab/ha)	(hab/dom)
Recife	**1.422.905**	**100**	**376.022**	**21.964,00**	**64,78**	**3,78**
RPA1-Centro	78.098	5,49	22.202	1.605,88	48,63	3,52
RPA2-Norte	205.986	14,48	52.383	1.429,95	144,05	3,93
RPA3-Nordeste	283.525	19,93	73.436	7.793,61	36,38	3,86
RPA4-Oeste	253.015	17,78	67.486	4.214,13	60,04	3,75
RPA5-Sudoeste	248.483	17,46	64.108	3.010,27	82,55	3,88
RPA6-Sul	353.798	24,86	96.407	3.901,79	90,68	3,67

Source: IBGE, EMPREL - 2005

hospital (h_{max}), with specified reliability of not be deficient in beds (α). With the characteristical data of the RPA – population distributed uniformly at the ZA and from the population density d (hab/ha or hab/km^2) of a RPA which contains the ZA and it determines its area A and, seeking to minimize, the expected value of the travel time $E(TV)$ establishes its geographic contour. In the case of $RPAs$ being too large, there can be distortion of the calculus of d. In doing so, each RPA is divided in ZAs disjointed so that their union constitutes the surface layer of the RPA considered.

It is important to point out that the geography of the internal contours of the ZAs should be approximately equidistant from hospital facilities, when the route on the map is made of the RPA. So, the division of $RPAs$ into ZAs is being considered and not division of the entire city as a whole. This methodology addressed is basically aimed to:

1) Maintain the same quota of calls throughout all the area to be divided ($d = constant$);

2) Each one of the $RPAs$ will contain a number of small hospitals (for example, varying from 1 to 5 hospitals) and consequently, be much easier to divide the ZAs with a more precise trace at the frontiers among all the assistance zones.

In the cases where the ZA is extensive, not satisfying the equation (1), the same one should be divided into sub-zones $SZAs$, where each one of the sub-zones correspond to an U_s service unit station and each U_s of the station will perform its service in that sub-zone with all the U_s leading their patients to the hospital of the considered ZA. In short, each sub-zone is a sub-space contained within the geometric space of the zone of assistance.

In considering the city as being a single ZA, with a single unit service station, the condition $E(TR) = E(TP)+E(TV) \leq TMR$ is not verified in most of the cases, due to the variables $TECSU_s$ (Time between Successive Calls from the U_s Service Units) and $TEESHU_s$ (Time between the Successive Entrances of the Patients in the Hospitalization using the Service U_s Unit) being small in relation to TS. Now, for the hospital beds, it is more viable, in economic terms, to evaluate the quantity of beds. Therefore, each individual of the ZA may be hospitalized in any hospital of the city. This form of dimensioning of the beds is much more advantageous than in relation to dimensioning the number of beds for the population of the j zones. To be precise, each individual just hospitalizing him/herself in the hospital corresponding zone, such that to satisfy the condition $\sum_{i=1}^{j} h_i = h$, where h_i and h are the population of the assistance zone and of the city, respectively. For these two forms of approach aiming towards quantifying/the demensioning of hospital beds, it must be guaranteed to both that there will not be a lack of beds. In this case, the reliability of not lacking beds ($\alpha = 1$) is considered. Therefore, in both forms of approach, the number of hospital beds n_L should be calculated in different forms. Note that

$$1): \quad n_{L_1} = h(1-p) + 4\sqrt{p(1-p)}\sqrt{\sum_{i=1}^{j} h_i} \quad \text{or} \quad n_{L_1} = h(1-p) + 4\sqrt{p(1-p)}\sqrt{h}.$$

$$2): \quad n_{L_2} = h(1-p) + 4\sqrt{p(1-p)}\sum_{i=1}^{j}\sqrt{h_i}.$$

As the term $\sqrt{\sum_{i=1}^{j} h_i} < \sum_{i=1}^{j}\sqrt{h_i}$ for $i > 1$, then $n_{L_2} > n_{L_1}$. To get an idea of the magnitude of the difference between n_{L_2} e n_{L_1}, consider, as an example, a city with 842.000 inhabitants, with the probability of an individual suffering an accident $(1-p) = 0.005$. Suppose that the dimensioning of the number of assistance zones is $j = 5$, being each h_i, for $i = 1, 2, \ldots, 5$, it is given by: $h_1 = 130.000$, $h_2 = 195.000$, $h_3 = 260.000$, $h_4 = 156.000$ and $h_5 = 101.000$. After the application of the equations of n_{L_1} and n_{L_2} it comes: $n_{L_1} = 4.210$ beds and $n_{L_2} = 4.468$ beds, bringing a reduction of 313 beds, with a economy of $5,77\%$, which represents an average sized hospital. Moreover, while it increases the strata of the urban area of the city in ZAs, the economic percentage increases.

Subsequently, the formatting model of an urbanregion integrated to transport system for pre-hospital assistance has the following characteristic: there is always an U_s station located at a hospital. This choice is mainly due to factors listed below:

a) Greater operational facility of the U_s by the hospital that can prepare itself to receive the injured patient, while the U_s leaves procuring the patient. Besides that, the current communication systems available radios on board computers and GPS are facilitating elements in the diagnostic relation between the doctor of the hospital and the paramedic of the U_s;

b) Low factor of U_s per hospital in the Brazilian cities;

c) With this model there is no problem of lack of beds in a given hospital;

d) With the creation of the satellite stations or with the continous circulation of the service units, the use of mileage completed by the U_s tends to increase hence diluting fixed costs, which represent more than 50% of the total cost. Making an analogy with urban transport by bus, each 2% of increase in the use of the bus there is a reflection in the reduction of the total cost of around 1% (Spreadsheet of Operational Cost of $STPP - RMR$, 2008);

e) The equations $E(TR) = E(TP) + E(TV) \leq TMR$ and $\overline{W} \equiv 0$ can be adopted for each of the ZAs.

If only one central for screening were established, with the sole purpose of the distribution of all the U_s, and if the U_s, of any one station of the ZA is distributed adequately throughout the whole corresponding zone, and if for any given call to the central screening to expedite the closest U_s available to the vicinity of the accident, you have as a response a greater improvement in the service provided by U_s due to none limitation of the service areas and at the same time a larger reduction in the TV from the maximum decentralization of the U_s in each ZA. The very screening central informs the U_s the nearest hospital with available beds.

The focus to be given needs limitations; chiefly, by the use of variables inherent to the communication factor. In these terms, the model should be taken into account that the U_s are fixed units of the stations and with a range of widespread assistance – service equity in the entire urban region.

After this brief insight into the focus of the distribution of U_s, it is necessary to study in a wide context the variables that interfere in equations (1) and (2).

3 Study of the variables travel time – TV

One of the relevant aspects in the planning of a logistic system is the definition of the location of the places that form a network of delivery services. These locations are, normally, associated with suppliers, warehouses, deposits and service assistance locations or any other elements of a network for which it is possible to demarcate its geographic position. The definition of the geographic position of these locations contributes, significantly, to reduce transport costs, to minimize time response, to maximize the user's satisfaction or optimization of some utility measure as to reflect the strategies to be adopted for the pre-hospital assistance service.

Therefore, one has to look for statistical methods of analysis which can solve the problems of geographical location of positions of a network of assistance, measuring in a robust manner the distances between these positions and looking to place them optimally under certain transportation restrictions, such as time and costs.

Initially, an important feature in the analysis of location problems is the trajectory restriction. When there are constraints of routes between two locations, and the paths are the arcs of the graph, the graph analysis normally represents the location problems under a transport system in an defined mesh, where they already have determined the possibilities for a route. Hence, the optimization of these paths in a graph is approached through the appropriate theory.

In everyday language, distance is the measure of the separation of two positions. The distance between two locations is measured by the length of the straight line segment that joins them. When referring to the distance between two points on the surface of the Earth, then the distance is the minimum length among the possible trajectory on the surface from a point and reaching the second (geodesy). In practical applications it is common to set the distance between two points on Earth as the length of the trajectory used by a certain means of transport. So, we refer to the distance from urban and road network distance, railroad or air distance. The distance is always a positive measure and has the property that the distance from point A to point B is identical to the distance from point B to point A.

The first step to the problem of the location of the positions, where the stations should be located, is the definition of the metric, which should be used to infer the distances. There are various metrics, although the ones used mostly are the Euclidean Metric (ME) and the Rectangular Metric or Metropolitan Metric (MM).

The Euclidean metric adopts the principle that the shortest path between two points is a straight line and the measurement of that is based on the geometry developed by Euclid. Despite being the ideal metric, because it deals with the real Euclidean spaces measures, and consequently presents excellent results in its applications. There are some computing inconveniences with the use of the Euclidean metric. The problem of using the transformed distance with this type of metric for distinct objects, is not always algorithmically easy to implement, neither is its computerized calculus efficient, since it involves the calculation of squares and roots. The second – MM – is much more dealt with in urban road networks being more collinear with the perpendicular strokes of the vectors which constitute the routes of the paths formed by streets and avenues. In this metric there is an interesting interpretation: when applied to positions of the discreet space, this metric assumes that, to go from point A to a point B, it is only possible if you can only go in the directions of the main axes of coordinate systems, where the area is defined (it is not permitted to go in the diagonal directions).

Consider a given ZA_i of average population density d. This zone represents the service area where the U_s station is situated, the one that is denominated U_s hospital station or service station. To make the approximation of the shape of the ZA_i known area A_i, could ever admit in a practical manner the representation of ZA_i in various formats of plain geometric figures; a circle, a square, quadrant of a geometrical circle, among others.

Here, it is the hospital facility that can occupy various positions inside the geometric figure. Consider v an average scalar speed of an U_s that must be evaluated and considered minimal due to the route of U_s being small, or happening, for example, during the "rush" hour. Denote by D is the random variable that explains the route of U_s from the station where the hospital is located (i) at any point (j) of the broad geometric figure ZA. Then, the distance D_{ij} can be calculated in two ways. The first by means of the Euclidean Metrical equation ME give by:

$$D_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}. \tag{3}$$

The second form through the MM equation given by:

$$D_{ij} = |X_i - X_j| + |Y_i - Y_j|. \tag{4}$$

Using equations (3) and (4), we can calculate the distance of the trajectory made by the U_s between nodes i and j. The first equation (ME) is easier to be calculated in the circle and quadrants figures (geometric circle), whereas the second (MM) is easier in the square figure, given that, the coordinates (X_i, Y_i) are also, independent random variables.

The equation that most closely approximate the actual route is the MM equation, since in topographic surveying it can always take the parallel Cartesian coordinate axes the directions of the main roads or streets in the area of the assistance zone ZA_i. One can also consider the ME equation corrected by a factor of adjustment - F_a, which is the function of the geometry of the streets of ZA_i and can be estimated from the comparison between travel times observed with the theoretical ones. In practice, the transport network rarely has a trajectory between two points in the form of a straight line or rectangular shape. Usually, the actual distances between points are greater than the Euclidean distance, because this tends to be the minimum distance. Thus, to infer the distances using the ME or MM, one attempts to adjust them to actual distances using a correction factor. This factor (Novaes, 1989) can be obtained by adjusting the points by means of minimum linear squares. So, given the positions A and B, its distance $D(A, B)$ can be explained through the equations of the type $E(D) = \hat{a} + \hat{b}E(D_{mm})$ or $E(D) = \hat{a} + \hat{b}E(D_{me})$. For the urban network of the city of So Paulo, the following equations explain the relationships between the effective distance and Euclidean or metropolitan distances.

a) $D = 0,81 + 1,366\ D_{me}$,

b) $D = 1,13 + 1,045\ D_{mm}$.

It should be noted that the adjustment factors of the urban network is larger than those estimated for a roadway network. This happens due mainly to the restrictions of traffic in the urban network and also by the strong urban intersections included in the urban network. As a consequence, the adjustment factors for the ME (F_{ae}) are always greater than or equal to the adjustment factor for MM (F_{am}). Therefore, from $D = (F_{ae})D_{me}$ and $D = (F_{am})D_{mm}$, we have $F_{am} \leq F_{ae}$.

In a work about "Strategies for Sinzing Service Territories", Rosenfield et al. (1989) mentions "that the costs of service of the units undertaking the service is an important factor which can be minimized thanks to the efficiency of the service". Encouraging the improvement of the service is to increase the life chances of the patient. However, its costs practically can not be ignored when compared with the increasing medical costs. Since each service territory – ZA has a resource (U_s) which serves as a service, the problem of determining areas of service – ZA also requires determining the location of the service station of the U_s, as well as, the number of installations of these stations. Since in each ZA more services are added, in this manner, the organization of the system becomes much less centralized, although the set costs deriving from the installations of these stations should rise. On the other hand, the service units (U_s) being closer to the assistance, the efficiency and efficacy indicators improve compensating the increase of the set costs. Therefore, decentralization contributes with fewer units (U_s) for the same quality of service.

Langevin and Soumis (1989) defined the width L of a given region by the equation $L = \sqrt{A} - Br$, where r is the distance from the center of the region (Post or service station or service post of the Hospital U_s) to the position where the assistance happens, A the area of the region and B a real number. Thus, for a given demand assumed to be uniform, the time travel TV is proportional to $r - \dfrac{L}{2}$ $= r\left(1 + \dfrac{B}{2}\right) - \dfrac{\sqrt{A}}{2}$ (Rosenfield et al., 1989). By integrating from 0 to R over a ring size $2\pi r dr$, the expected value of travel time is of the form $KR - B$, where K is a constant and R proportional to the square root of the area A. Consequently, the travel time is not strictly related to the square root. However, when evaluating the equation the results suggest a relatively small value of B, i.e., asymptotically tends to zero.

Although the constant of proportionality is a function of square root, it will vary depending on the geometric shape of the ZA service, for instance, circular versus square. However, the distance from the service station of the ZA to the assistance position of this service, or point where the relief will be made to the injured by U_s within the ZA, can always be calculated by the ME or the MM (Larson, 1974).

The total hours worked per day on each trip can be defined in terms of number of visits to each ZA, as well as, to the total of assistance in all of the areas (ZAs). Observe that the total number of drivers can exceed the number of trips in the case of utilization of the U_s not being made.

Burns et al. (1985) established for a given assistance service zone an average travel time value for the assistance within the zone. This expected value of time is given by

$$E(TV) = K_1\sqrt{\frac{A}{n_s}} \quad or \tag{5}$$

$$E(TV) = K_2\sqrt{\frac{A}{M}}, \tag{6}$$

where K_1 and K_2 are constants, A denotes the coverage area of the entire geographic area of the ZA, n_s the number of U_s at the stations and M the number of visits.

After all the citations that address the problem of defining the equations of TVs interpreters, we now require a study of $E(TV)$ based on a individualized methodology in accordance with the different distances to be considered with the format of the ZA. For doing that, D is a random variable that represents those distances. As a result, $E(TV) = \dfrac{E(D)}{v}$ and $V(TV) = \dfrac{V(D)}{v^2}$, where v is the constant average velocity and h is the population that follows a distribution uniformly spread in the assistance zone.

If the study demands assistance restrictions, that is, the private hospitals contracts with the NHS, their U_s only attend their own clients. Then, one can calculate the population density (d) of a particular area of ZA assistance, based on the population θh, where θ represents the probability of an inhabitant of the ZA be someone insured by the NHS. One observes that θh has a known distribution probability, because it is easily estimated for each city. Therefore, its probability density function f the cumulative distribution function F and the coefficient of variation c can be determined. Next, we present the calculations of the average $E(D)$, $V(D)$ and c of the random variable which explains D, for the some geometric shapes of ZA.

4 Study of the random variable D based on the geometric of ZA

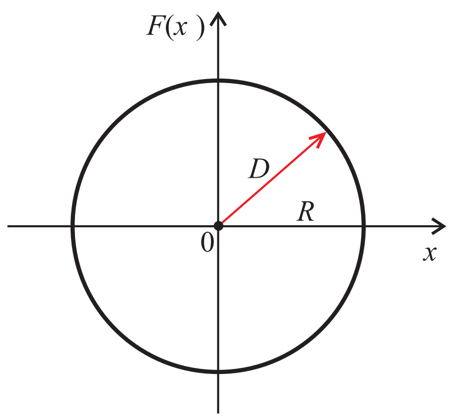

Figure 1 - The ZA with circle format with the hospital U_s station in its center.

The distribution function of the random variable D is given by

$$F(x) = \frac{x^2}{R^2} \quad \text{to} \quad 0 \leq x < R \quad \text{and} \quad F(x) = 1 \quad \text{to} \quad R \leq x < \infty. \tag{7}$$

Hence, $f(x) = \dfrac{2x}{R^2}$. We obtain $E(D)$, $V(D)$ and c as

$$E(D) = \int_0^R xf(x)dx \quad \text{or} \quad E(D) = \int_0^R x\frac{2x}{R^2}dx = \frac{2}{3}R \quad \text{or}$$

$$E(D) = \frac{2}{3}\sqrt{\frac{A}{\pi}} = 0,376\sqrt{A}. \tag{8}$$

$$V(D) = \int_0^R x^2\frac{2x}{R^2}dx - (0,376\sqrt{A})^2 \quad \text{or} \quad V(D) = 0,0178\ A. \tag{9}$$

$$c = \frac{\sqrt{0,0178\ A}}{\sqrt{(0,376)^2\ A}} = 0,354. \tag{10}$$

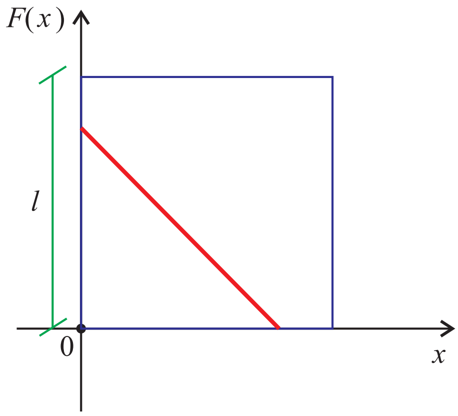

Figure 2 - The ZA with a Square Format with the Hospital U_s Station in one of the Vertexes.

The metric calculation of D involves the construction of a distribution function $F(x)$ between $0 \leq x \leq l$ e $l \leq x \leq 2l$. So, for the calculations of the $F's$ we consider the halves of the comprehended areas between the intervals $0 \leq x \leq l$ e $l \leq x \leq 2l$. Thus, for the square of the area $A = l^2$, we have

$$equao(10)F(x) = \frac{x^2}{2l^2}, \quad 0 \leq x \leq l \ \ and \quad F(x) = \frac{2-(2l-x)^2}{2l^2}, \quad l \leq x \leq 2l. \tag{11}$$

The density function of D follows by deriving $F(x)$ in for each of the intervals. Remember that, area in the interval $0 \leq x \leq l$ is $F(D \leq x) = 0,5$ and the area in F($l \leq x \leq 2l$), which is equal to $\left(\frac{1}{l^2} - \frac{2-l^2}{2l^2}\right) = 0,5$. Like this, F is a true cumulative function.

For the calculations of $E(D)$, $V(D)$ and c we have to calculate the integrals below:

$$E(D) = \int_0^l x\frac{x}{l^2}dx + \int_l^{2l} x\left(\frac{2l-x}{l^2}\right)dx = l \quad \text{or} \quad E(D) = \sqrt{A}. \tag{12}$$

$$V(D) = \int_0^l x^2\frac{x}{l^2}dx + \int_l^{2l} x^2\left(\frac{2l-x}{l^2}\right)dx - l^2 = \frac{14l^2}{12} - l^2 = \frac{A}{6}. \tag{13}$$

$$c = \frac{\sqrt{\frac{1}{6}A}}{\sqrt{A}} = 0,408. \tag{14}$$

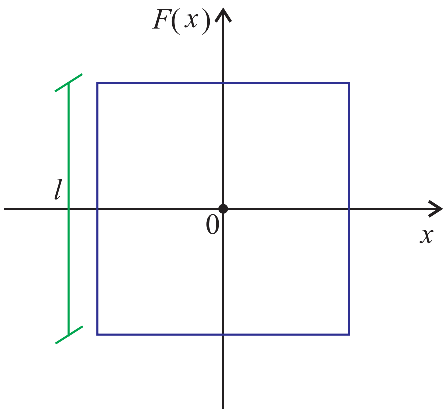

Figure 3 - The ZA with the Square Format of side l, with the Hospital U_s Station in its Center.

For the Figure 3, the metric is calculated based on the distribution function of D given by $F(x) = \frac{1}{2}\left(\frac{x^2}{2l^2}\right)$ to $l \leq x \leq 2l$. Consequently, the values of $E(D)$, $V(D)$ and c are derived based on the equations below:

$$E(D) = \frac{1}{2}\int_0^l x\frac{x}{l^2}dx + \frac{1}{2}\int_l^{2l} x\left(\frac{2l-x}{l^2}\right)dx = \frac{1}{2}l \quad \text{or} \quad E(D) = \frac{1}{2}\sqrt{A}. \tag{15}$$

$$E(D^2) = \int_0^l \frac{x^2}{2}\frac{1}{2}\left(\frac{x}{l^2}\right)dx + \int_l^{2l} \frac{x^2}{2}\frac{1}{2}\left(\frac{2l-x}{l^2}\right)dx \quad \text{or} \quad E(D^2) = \frac{1}{4}\left(\frac{14l^2}{12}\right). \tag{16}$$

$$V(D) = \frac{1}{4}\left(\frac{14l^2}{12}\right) - \frac{1}{4}l^2 \quad \text{or} \quad V(D) = \frac{1}{24}l^2 = \frac{1}{24}A. \tag{17}$$

$$c = \frac{\sqrt{\frac{A}{24}}}{\sqrt{\frac{A}{4}}} = \sqrt{\frac{1}{6}} = 0,408. \tag{18}$$

The calculation of $E(D)$ and $V(D)$ is immediate. By simple substituting in equations (11) and (12) the value of A by $\frac{A}{4}$.

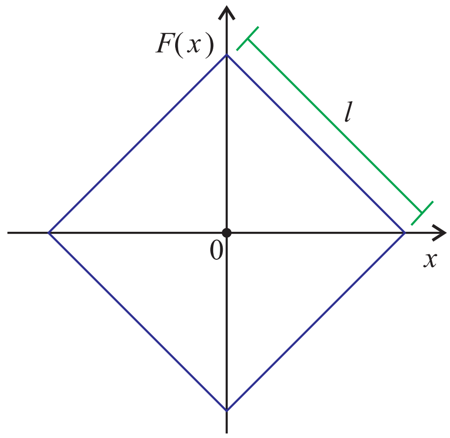

Figure 4 - ZA with Format of Square with Hospital U_s Station in its Center, Rotated $\angle = 45$ degrees.

The half-diagonal of the square is $l\frac{\sqrt{2}}{2}$. Hence, the distribution function of the random variable D is $F(x) = \frac{(x\sqrt{2})\,(x\sqrt{2})}{l^2}$ to $0 \leq x \leq l\frac{\sqrt{2}}{2}$ and zero for other values of D. The density function is $f(x) = \frac{4x}{l^2}$. In this manner, the mean, the variance and dispersion coefficient are calculated from the following expressions:

$$E(D) = \int_0^{l\frac{\sqrt{2}}{2}} x\frac{4x}{l^2}dx = \frac{\sqrt{2}}{3}l = \frac{\sqrt{2}}{3}\sqrt{A}. \tag{19}$$

$$E(D^2) = \int_0^{l\frac{\sqrt{2}}{2}} x^2\frac{4x}{l^2}dx = \frac{l^2}{4} = \frac{A}{4}. \tag{20}$$

$$V(D) = \frac{l^2}{4} - \frac{2l^2}{9} = \frac{l^2}{36} = \frac{A}{36}. \tag{21}$$

$$c = \frac{\sqrt{\dfrac{A}{36}}}{\dfrac{\sqrt{2A}}{3}} = \frac{1}{2\sqrt{2}} = 0,354. \tag{22}$$

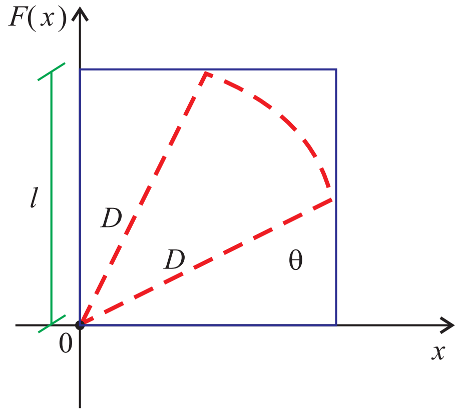

Figure 5 - The ZA with Square Format with Hospital U_s Station in the Vertexes and D by the ME.

For the interval $0 \leq x \leq l$, the distribution function of the random variable D is $F(x) = \frac{\pi}{4}\frac{x^2}{l^2}$; and, for the interval $l \leq x \leq l\sqrt{2}$, the metric is $D = \left(1 - \frac{\pi}{4}\right) l\sqrt{2}$. The calculation of $E(D)$, $V(D)$ and c are presented as follows:

$$E(D) = \frac{\pi}{4}\int_0^l x\frac{2x}{l^2}dx + l\sqrt{2}\left(1 - \frac{\pi}{4}\right) \quad \text{or } E(D) = 0,8273\sqrt{A}. \tag{23}$$

$$E(D^2) = \frac{\pi}{4}\int_0^l x^2\frac{2x}{l^2}dx + \left(l\sqrt{2}\right)\left(l\sqrt{2}\right)\left(1 - \frac{\pi}{4}\right) \quad \text{or}$$
$$E(D^2) = 0,8225\ A. \tag{24}$$

$$V(D) = 0,8225A - \left(0,8273\sqrt{A}\right)^2 = 0,1381\ A. \tag{25}$$

$$c = \frac{\sqrt{0,1381\ A}}{0,8273\sqrt{A}} = 0,4492. \tag{26}$$

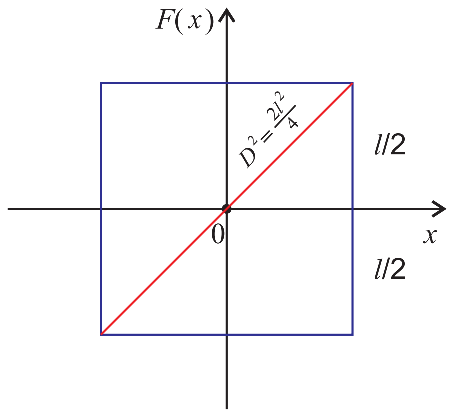

Figure 6 - The ZA with Square Format with Hospital U_s Station in the Center and D by the ME.

A extremely satisfactory result to obtain the values of $E(D)$, $E(D^2)$, $V(D)$ is to use in equations (22), (23) and (24) the quantity $\frac{A}{4}$ instead A. The results are presented below:

$$E(D) = 0,8273\sqrt{\frac{A}{4}} = 0,41365\sqrt{A}. \tag{27}$$

$$E(D^2) = 0,8225\left(\frac{A}{4}\right) = 0,20562\ A. \tag{28}$$

$$V(D) = 0,20562\ A - (0,41365\sqrt{A})^2 \quad \text{or} \quad V(D) = 0,03451\ A. \tag{29}$$

$$c = \frac{\sqrt{0,03451A}}{0,41365\sqrt{A}} = 0,4492. \tag{30}$$

Consider the circle circumscribed to the square of side $R\sqrt{2}$. The distribution function is given by $F(x) = \frac{2}{\pi}\frac{x^2}{R^2}$ and the metric $D = R\sqrt{2}\left(1-\frac{2}{\pi}\right)$. The relationship between the areas of the square and of the circle is $\frac{(R\sqrt{2})^2}{\pi R^2} = \frac{2}{\pi}$. The expressions for $E(D)$, $V(D)$ and c are:

$$E(D) = \frac{2}{\pi}\int_0^R x\frac{2x}{R^2}dx + \left(1-\frac{2}{\pi}\right)\ R\sqrt{2} \ \text{ or } E(D) = 0,938\sqrt{A}. \tag{31}$$

$$E(D^2) = \frac{2}{\pi}\int_0^R x^2\frac{2x}{R^2}dx + \left(R\sqrt{2}\right)\left(R\sqrt{2}\right)\left(1-\frac{2}{\pi}\right) \quad \text{or} E(D^2) \quad = 1,0445\ A. \tag{32}$$

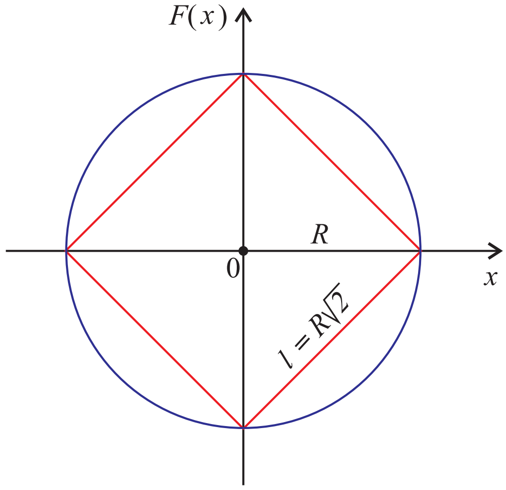

Figure 7 - The ZA with Format of a Circle of radius R with Hospital U_s Station in the Center and D by the MM.

$$V(D) = 1,0445A - \left(0,938\sqrt{A}\right)^2 = 0,1646\ A. \tag{33}$$

$$c = \frac{\sqrt{0,1645\ A}}{0,938\sqrt{A}} = 0,4325. \tag{34}$$

Table 2 - Analysis of the Metrics of D as a Geometry Function of the ZA.

Geometry of the ZA	**Location of the Station**	**Metric**	$E(D)$	$V(D)$	c
Radius R Circle	$*$	ME	$0,376\sqrt{A}$	$0,0178\ A$	$0,354$
Square of side l	$**$	MM	$\sqrt{A}$	$0,1666\ A$	$0,408$
Square of side l	$*$	MM	$0,500\sqrt{A}$	$0,0416\ A$	$0,408$
Sq. of side $l\angle 45$ degrees	$*$	MM	$0,4714\sqrt{A}$	$0,0277\ A$	$0,354$
Square of side l	$**$	ME	$0,8273\sqrt{A}$	$0,1381\ A$	$0,4492$
Square of side l	$*$	ME	$0,4136\sqrt{A}$	$0,0345\ A$	$0,4492$
Radius R Circle	$*$	MM	$0,938\sqrt{A}$	$0,1646\ A$	$0,4325$

$*$ Hospital U_s Station in the Center $**$ Hospital U_s Station in the Vertexes

The figures in Table 2 show that for calculating D by MM, the geometric form for the ZA, which responds with a smaller $E(D) = 0,4714\sqrt{A}$, smaller $V(D)$ and c is the square of side l with a spin of $\angle 45$ degrees in relation to the horizontal axis and hospital U_s station located in its center. We observe that, for any metric, the figure of the square of side l with the hospital station in the center responds also with a maximum value for $E(D) = 0,500\sqrt{A}$. Thus, it can be concluded that the geometric figure of $Min\ (E(D))$ also has a $Min\ (D_{max})$. So, if you try to minimize $E(D)$, you are minimizing at the same time D_{max}.

To evaluate the accuracy that any geometric figure of $Min(E(D))$ also has a $Min(D_{max})$, it is necessary to study a general geometric figure for the ZA. Be it, a square (Q) of side l with a spin of $\angle 45$ in relation to the horizontal, as shown in Figure 7. Therefore, for any Figure G, we can show that $E(D_Q) = Min\ (E(D_G))$.

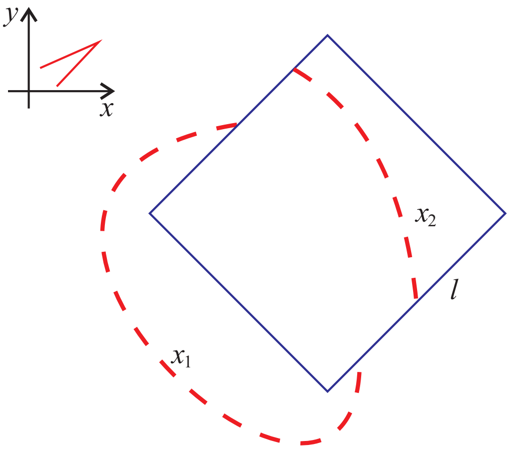

Figure 8 - Analysis of the $E(D_G)$ for the Figures at the Square Form x General Form.

The proof is based on the MM. Consider that the value of $E(D_Q)$ is not the minimum, so we can define a boundary region covering part of the square of side l, similar to the one presented in Figure 8. Based on this Figure G we add to the square a set of points X_1 and then remove another set of points X_2, to satisfy the inequality $E(D_G) \leq E(D_Q)$. As the area of Figure G must be equal to the area of the square $(A_Q = A_G)$, so the vector $\overrightarrow{BX_1} = \overrightarrow{BX_2}$ or $|X_1| = |X_2|$, to whichever point B is inside Figure G. Consequently, the value expected $E(D_G)$ can be calculated by a weighted mean, that is:

$$E(D_G) = E(D|X_1)\frac{|X_1|}{A_G} + E(D_Q)\frac{A_Q}{A_G} - E(D|X_2)\frac{|X_2|}{A_G}. \tag{35}$$

As shown in Table 2, for the square of side l, for MM with hospital U_s station in the center, the maximum value is $E(D) = 0,5\sqrt{A}$. So, $E(D|X_1) \geq 0,5\sqrt{A}$ and $E(D|X_2) \leq 0,5\sqrt{A}$. Substituting in equation $E(D_G)$, we have $E(D_G) \geq E(D_Q)$ setting up real algebraic nonsense by inequality $E_G(D) \leq E_Q(D)$. Thus, $E_Q(D) = Min\ (E_G(D))$ is confirmed.

You can use the same procedure to put for a circle with a station in its center and considering the ME of D. This circle provides the $Min\ (E(D))$, having $V(D)$ and c minimum and $Min\ (D_{max})$ as well. For this circle, the value of $c = 0,354$, which is the same in the MM for the square of side l with spin $\angle 45$ degrees to the horizontal axis and the hospital U_s station located in its center.

In short, knowing the area of a ZA and choosing the MM to determine its contour attempts a format for the ZA close to a square with a $\angle 45$ degree rotation in relation to the horizontal axis with the Cartesian parallel axes to the streets or main roads of the RPA that contain the ZA or SZA contained in ZA.

4.1 The RPA division of area A and populational density d in N zones of assistance

Consider the surroundings of an urban area A, one RPA containing several districts with constant population density d. When dividing this RPA in N zones of assistance ZA_j, $j = 1, 2, \ldots, N$, the objective is always to obtain a minimum and a maximum $E(D_j)$ for each ZA. Each ZA_j contains service unit stations (U_s).

To be an individual of the RPA; accepted that every individual i has the same probability of requesting or calling the U_s service unit in spite of its location in the RPA. Therefore, to calculate the $Min\ (E(D))$ it is necessary to obtain the critical points of the function $W(A, \Theta) = E(D) + \Theta\left(\sum_{j=1}^{N} A_j - A\right)$, that is: $\dfrac{\partial W}{\partial A_j} = 0$ and $\dfrac{\partial W}{\partial \Theta} = 0$, where Θ is denominated the Lagrange multiplier.

$$E(D) = \sum_{j=1}^{N} E(D_j)\frac{A_j}{A} = \sum_{j=1}^{N} \frac{\sqrt{2}}{3}\sqrt{A_j}\frac{A_j}{A} = \frac{\sqrt{2}}{3}\sum_{j=1}^{N}\frac{(A_j)^{1,5}}{A}.$$

It is noted that the $Min(E(D)$ is $Min(\sum_{j=1}^{N}(A_j)^{1,5}$. So,

$$\frac{\partial W}{\partial A_j} = \frac{\sqrt{2}}{3}\frac{3}{2}\frac{1}{A}(A_j)^{0,5} + \Theta = 0.$$

$$\frac{\partial W}{\partial \Theta} = \left(\sum_{j=1}^{N} A_j - A\right) = 0.$$

Remember that the points that minimize W miminize $E(D)$. Where at, $Min(W)$ is obtained to $A_1 = A_2 = ... = A_j = \dfrac{A}{N}$. Subsequently,

$$E(D) = \frac{\sqrt{2}}{3}N\left(\frac{A}{N}\right)^{0,5}\left(\frac{A}{N}\right)\left(\frac{1}{A}\right) = \frac{\sqrt{2}}{3}\sqrt{\frac{A}{N}}. \tag{36}$$

$$E(D^2) = \sum_{i=1}^{h} E(D_i^2)\frac{A_i}{A} = \sum_{i=1}^{h}\frac{i}{2d}\frac{A_i}{A} = \frac{h}{4\ d}\frac{A_i}{A} = \frac{A}{4\ N}.$$

$$V(D) = E(D^2) - E(D)^2 = \frac{A}{4\ N} - \frac{2\ A}{9\ N} = \frac{A}{36\ N} = 0,027\frac{A}{N}. \tag{37}$$

This result represents the most favorable situation for the minimization of the expected value of the metric D. In short, given a region defined by an RPA, this will give a minimum for $E(D)$ if the region of RPA is divided in N zones of assistance of the same area A and with the U_s service stations located in the center of the zones with formats equal to a square of side $l\sqrt{2}$ and rotated by $\angle 45$ degrees in relation to the horizontal.

4.2 Distance between adjacent points

A typical problem of logistics is to evaluate the distance between points near a region that are distributed by a Poisson process. This assessment occurs in the design of public emergency services, maintenance crews and police patrol techniques. As seen earlier, this distance is calculated according to the Euclidean and metropolitan metric.

Considering that each inhabitant is an exclusive user of the nearest service station we can estimate an superior limit for $E(D)$ assuming that the N stations are distributed randomly in the region (RPA). This is because we should desire that there are few posts distributed with the worst $E(D)$ if they were not randomly distributed. Thus the probability of finding k posts in the region R of area $A(R)$ is:

$$P(X(R) = k) = \frac{\left(\frac{N\ A(R)}{A}\right)^k \exp\left(\frac{-NA(R)}{A}\right)}{k!}, \quad to \quad k = 0, 1, ... \quad and \quad A(R) \geq 0.$$

For the metropolitan metric, we suggest the following steps to estimate the metric:

a) Consider an incident point in the region R of (X, Y) coordinates;

b) Construct a square rotated at $\angle 45$ degrees, with the distance from its center to one of the equal vertices to r. The area of this square defineed of the region R is $A(R) = (r\sqrt{2})(r\sqrt{2}) = 2\ r^2$;

c) As d the constant population density of the ZA, A the total area of the RPA and N the number of ZAs, then the probability that there be exactly k service stations within the square where your side is $r\sqrt{2}$ can be written as:

$$P(X(R) = k) = \frac{\left(\frac{N\ 2\ r^2}{A}\right)^k \exp\left(\frac{-N2\ r^2}{A}\right)}{k!}.$$

Therefore, a probability of there not being a station at $X(R)$ is $P(X(R) = 0) = \exp\left(\frac{-N2\ r^2}{A}\right)$.

d) Consider D the distance from the midpoint of the square to the nearest station. Then the cumulative distribution function of D is $P(D \leq r) = 1 - P(D > r)$, but $P(D > r) = P(X(R) = 0)$. Then, $F(r) = 1 - \exp\left(\frac{-N\ 2\ r^2}{A}\right)$ for $r \geq 0$. To obtain the statistics of D it is necessary to calculate the density function of D, which represents the derivative of $F(r)$ in relation to r, that is,

$$f_D(r) = \frac{4N\ r}{A} \exp\left(\frac{-N\ 2\ r^2}{A}\right), \quad to \quad r \geq 0. \tag{38}$$

The density function $f_D(r)$ is known as a Rayleigh Distribution with parameter $\frac{1}{\sqrt{\frac{4\ N}{A}}} = \sqrt{\frac{A}{4N}}$. So to a certain inhabitant of the ZA, which calls the service station nearest to their coordinates (X, Y) are independent normally distributed with zero mean and variance $\frac{A}{4N}$. The route of the U_s for their assistance is defined by the euclidean metric $\sqrt{X^2 + Y^2}$, with the system of orthogonal axes in the duty station. From $f_D(r)$ of Rayleigh comes:

$$E(D) = \sqrt{\frac{A}{4N}}\sqrt{\frac{\pi}{2}} = 0,625\sqrt{\frac{A}{N}}. \tag{39}$$

$$V(D) = E(D^2) - E(D)^2 = 2\frac{A}{4\ N} - \frac{A}{4N}\frac{\pi}{2} = 0,108\frac{A}{N}. \tag{40}$$

$$c = \frac{\sqrt{0,108\frac{A}{N}}}{0,625\sqrt{\frac{A}{N}}} = 0,525. \tag{41}$$

As an illustration we have the following problem: "Calculate the mean distance and standard deviation of a point of the zone of area assistance A to a closer service station for a service of pre-hospital assistance made by U_s in a more suitable urban environment for the metropolitan metric, where the density of calls from the U_s is $d = 0,5$ calls per km^2. Consider also the adjustment coefficient F_a of the distance equal to $1,18$". The values for $E(D)$ and $V(D)$ are:

$$E(D) = F_a\sqrt{\frac{A}{4N}}\sqrt{\frac{\pi}{2}} = 1,18\left(0,625\sqrt{\frac{1}{d}}\right) = 1,043\ km.$$

$$V(D) = 1,18^2\left(0.108\frac{A}{N}\right) = 1,18^2\left(0,108\frac{1}{d}\right) = 0,300\ km^2.$$

$$\sigma = 1,18\sqrt{0,216} = 0,548\ km$$

On the Rayleigh distribution, consult (Hines et al., 2006). The values of $E(D)$, $V(D)$ and c based on the Rayleigh distribution represent the most disfavorable situation. Making $N = 1$ – a single ZA – the equation (35) is the equation (18). The result of the equation (18) is reduced in $5,72\%$ compared to the result of the equation (14). Now, making the comparison of the equation (35) to equation (38) you should, for the optimal distribution of the posts that contain the $U_s s$ in a

RPA, its expected value $E(D)$ decreases from 24,76% consider to the expected value $E(D)$ obtained when the posts are randomly distributed, and admitting that the user inhabitant of the ZA is always looking for the nearest station.

Since it's pretty unlikely to optimally divide a RPA and considering that few situations are worse than the completely random, it can thus be considered for the calculation of $E(D)$ and $V(D)$, the equations of a queuing model of type $M/G/n_s$ for the U_s located at a hospital post. Thus, the equations of the expected value $E(D) = 0,50\sqrt{A}$ and of the variance $V(D) = \dfrac{A}{36}$ are applied considering that the ZA is close to a rotated square $\angle 45$; and, for N = 1, the equations to be used are (38) and (39) for $E(D)$ and $V(D)$, respectively.

After this condensed analysis by means of the equations mentioned above it was concluded, that one should always seek to divide a region type RPA in assistance zones ZA close to a rotated square of $\angle 45R$ from in relation to the horizontal axis, with boundaries or internal designs equidistant from the hospitals so that the resident when injured be attended by the nearest service station. Note, that even increasing in 100% the number of zones ZA, the reduction that is obtained for $E(D)$ in (35) and (38) is about 25%.

As seen earlier the search to maximize efficiency for the assistance service using pre-hospital U_s will be given according to the assumptions explained by the equations below:

$$E(TR) = E(TP) + E(TV) \leq TMR \tag{42}$$

and

$$\overline{W} \equiv 0, \tag{43}$$

in which TMR represents the maximum response time specified for any assistance area, according to the geometric format that leads to a minimum $E(TR)$.

Thus, the defining equation of the criterion of the U_s efficiency due to the ZA geometric formatting is defined as follows, considering the time in minutes, the A area in km^2 and v in km/h.

1) If the geometry of the ZA is close to a rotated square of $\angle 45$ degrees in relation to the horizontal axis, then:

$$[E(TP) + E(TV)] \leq TMR \quad \text{or} \quad [TMR - E(TP)]^2 \geq \left(\frac{E(D)}{v}\right)^2. \tag{44}$$

$$[TMR - E(TP)]^2 \geq \left(\frac{E(D)}{v}\right)^2. \tag{45}$$

$$[TMR - E(TP)]^2 \geq \left(\frac{0,4714\sqrt{A}}{v}\right)^2. \tag{46}$$

$$A \leq [TMR - E(TP)]^2 \frac{v^2}{800}. \tag{47}$$

2) For any geometry of the ZA,

$$[E(TP) + E(TV)] \leq TMR \quad \text{or} \quad [TMR - E(TP)]^2 \geq \left(\frac{E(D)}{v}\right)^2. \tag{48}$$

$$[TMR - E(TP)]^2 \geq \left(\frac{E(D)}{v}\right)^2. \tag{49}$$

$$[TMR - E(TP)]^2 \geq \left(\frac{0,625\sqrt{A}}{v}\right)^2. \tag{50}$$

$$A \leq [TMR - E(TP)]^2 \frac{v^2}{1410}. \tag{51}$$

The equations of efficiency suggest two forms of treatment, when formatting the ZAs. The first is to consider the TMR as being the fixed value for all the ZAs. In this manner the inherent costs to the dimensioning of the U_s units would increase owing chiefly to the more distant zones from the urban center. The second suggests considering the TMR as being a variable. This approach is conditioned to the cluster analysis among various $RPAs$ and ZAs, quantifying their similarities, through measures such as distance function, similarity coefficient and correlation coefficient.

CORDEIRO, D.; CORDEIRO, G. M.; LIMA NETO, O. Modelo de formatação de uma região urbana integrado ao sistema de transporte para atendimento pré-hospitalar. *Rev. Bras. Biom.,* Lavras, MG, v.34, n.1, p.107-132, 2016.

▪ ABSTRACT: O atendimento pré-hospitalar móvel é um serviço de assistência individualizada e especializada, fora dos estabelecimentos hospitalares. O intuito desse tipo de serviço é a maximização dos atendimentos visando à manutenção da vida. Esse tipo de atendimento tem como finalidade chegar ao usuário indivíduo acidentado em uma determinada região R no menor tempo resposta após a ocorrência do evento. O meio de transporte das remoções dos usuários para os estabelecimentos hospitalares feito por uma unidade de servio U_s. O perfil da mortalidade se alterou ao longo das últimas décadas, tanto no Brasil, quanto no mundo. Se por um lado, a melhoria das condições sanitárias e os progressos da medicina reduziram as mortes por vários tipos de doenças, a massificação do automóvel e a violência urbana, dentre outros fatores, criaram ou acentuaram urgências médicas provenientes dos traumas, principalmente motivados por acidentes de trânsito. Dessa maneira, o aumento substancial na curva do valor esperado dos atendimentos ao longo do tempo é fato notório. Para o atendimento pré-hospitalar pode-se tratar uma cidade como uma única região urbana de atendimento ou estratificá-la em zonas urbanas de atendimentos. Para o primeiro caso, uma única estação ou posto serve como central de triagem, o local é usado como distribuidor das U_s e, nesse caso, o paciente é removido para o estabelecimento hospitalar mais próximo do local onde se encontra o usuário do sistema (acidentado). O segundo caso, em cada zona de atendimento (ZA) é alocado um estabelecimento hospitalar, tais como, público e privado do município e do estado conveniados com o Sistema Único de Saúde (SUS). Desse modo, a modelagem a ser abordada para atendimento numa região a representaç ao de toda a área urbana da cidade e as zonas urbanas de atendimentos (ZA) funcionam como elementos de áreas da região que contém uma organização hospitalar. Em suma, cada uma das zonas de atendimentos é definida em função da minimização do tempo resposta (TR) tempo entre a chamada da U_s e sua chegada ao paciente. Esse é o objetivo único para garantir a vida do acidentado. Dessa maneira, tem-se que buscar métodos estatísticos de análise que resolvam os problemas de localização geográica dos pontos de uma rede de atendimentos, medindo de forma robusta as distâncias entre esses pontos e procurando situá-los de forma otimizada sob certas restrições de transporte, tais como tempos e custos.

▪ KEYWORDS: Zona de atendimento; sistema hospitalar; rede de integrao; posto de servio; métricas; distribuio de Rayleigh.

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Recebido em 02.03.2015.

Aprovado após revisão em 05.09.2015.